| $i-1 / j-1$ | $i / j-1$ | $i+1 / j-1$ |
| :---: | :---: | :---: |
| $i-1 / j$ | $i / j$ | $i+1 / j$ |
| $i-1 / j+1$ | $i / j+1$ | $i+1 / j+1$ |



| 166 | 89 | 72 | 89 | 79 | 82 | 139 | 84 | 22 | 44 | 99 | 131 | 156 | 183 | 193 | 168 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 102 | 49 | 39 | 51 | 62 | 70 | 72 | 31 | 22 | 56 | 124 | 161 | 180 | 205 | 212 | 190 |
| 70 | 34 | 31 | 62 | 65 | 107 | 114 | 70 | 56 | 72 | 114 | 143 | 180 | 205 | 207 | 171 |
| 44 | 29 | 24 | 44 | 124 | 92 | 67 | 70 | 56 | 46 | 60 | 151 | 226 | 188 | 141 | 99 |
| 31 | 29 | 7 | 14 | 84 | 39 | 14 | 27 | 24 | 44 | 114 | 126 | 236 | 219 | 107 | 67 |
| 24 | 39 | 22 | 9 | 12 | 27 | 19 | 29 | 19 | 54 | 109 | 89 | 183 | 255 | 193 | 97 |
| 24 | 27 | 24 | 17 | 17 | 39 | 75 | 102 | 82 | 94 | 126 | 141 | 168 | 231 | 231 | 178 |
| 22 | 22 | 27 | 36 | 19 | 29 | 114 | 121 | 121 | 141 | 168 | 168 | 193 | 207 | 200 | 166 |
| 19 | 22 | 36 | 41 | 17 | 22 | 60 | 75 | 116 | 107 | 87 | 161 | 193 | 188 | 156 | 129 |
| 17 | 24 | 56 | 54 | 17 | 14 | 94 | 158 | 97 | 79 | 82 | 89 | 126 | 134 | 119 | 111 |
| 14 | 27 | 27 | 9 | 12 | 19 | 84 | 156 | 77 | 75 | 102 | 60 | 79 | 104 | 97 | 84 |
| 12 | 17 | 9 | 2 | 19 | 29 | 54 | 114 | 87 | 89 | 109 | 79 | 79 | 67 | 72 | 94 |
| 14 | 17 | 14 | 41 | 77 | 99 | 131 | 178 | 151 | 153 | 168 | 139 | 121 | 75 | 65 | 82 |
| 27 | 36 | 72 | 129 | 156 | 158 | 163 | 200 | 180 | 175 | 195 | 180 | 136 | 49 | 17 | 17 |
| 29 | 54 | 134 | 180 | 188 | 180 | 158 | 168 | 195 | 190 | 183 | 146 | 107 | 22 | 0 | 2 |
| 24 | 36 | 97 | 148 | 163 | 175 | 171 | 161 | 168 | 173 | 143 | 99 | 67 | 12 | 7 | 7 |



| 101 | 86 | 64 | 65 | 72 | 84 | 79 | 61 | 43 | 61 | 102 | 141 | 169 | 188 | 191 | 190 |
| :---: | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 85 | 72 | 57 | 61 | 74 | 87 | 85 | 67 | 50 | 67 | 104 | 143 | 171 | 191 | 192 | 190 |
| 54 | 46 | 40 | 55 | 75 | 85 | 77 | 62 | 53 | 67 | 103 | 148 | 182 | 193 | 179 | 170 |
| 39 | 33 | 30 | 50 | 70 | 78 | 66 | 55 | 51 | 65 | 96 | 150 | 186 | 189 | 156 | 132 |
| 32 | 27 | 24 | 37 | 49 | 53 | 42 | 36 | 41 | 58 | 88 | 143 | 185 | 194 | 151 | 117 |
| 29 | 25 | 20 | 22 | 28 | 36 | 41 | 43 | 52 | 74 | 99 | 143 | 183 | 202 | 175 | 145 |
| 26 | 25 | 24 | 20 | 22 | 39 | 61 | 75 | 84 | 101 | 121 | 149 | 181 | 206 | 195 | 177 |
| 22 | 24 | 28 | 26 | 26 | 43 | 70 | 96 | 106 | 115 | 132 | 156 | 183 | 196 | 187 | 176 |
| 21 | 27 | 35 | 33 | 27 | 42 | 76 | 106 | 112 | 110 | 120 | 140 | 162 | 168 | 156 | 146 |
| 20 | 26 | 32 | 29 | 22 | 37 | 75 | 101 | 104 | 91 | 93 | 108 | 126 | 132 | 124 | 116 |
| 18 | 22 | 25 | 22 | 19 | 38 | 80 | 102 | 103 | 88 | 84 | 89 | 90 | 97 | 98 | 96 |
| 16 | 16 | 18 | 23 | 34 | 58 | 96 | 114 | 120 | 112 | 108 | 104 | 89 | 84 | 82 | 82 |
| 20 | 24 | 37 | 57 | 78 | 98 | 125 | 139 | 147 | 145 | 143 | 134 | 102 | 75 | 59 | 57 |
| 29 | 44 | 75 | 110 | 134 | 145 | 159 | 169 | 176 | 176 | 169 | 152 | 108 | 65 | 36 | 30 |
| 34 | 56 | 98 | 140 | 164 | 168 | 170 | 173 | 178 | 178 | 164 | 139 | 90 | 46 | 14 | 8 |
| 35 | 62 | 108 | 151 | 172 | 172 | 168 | 170 | 175 | 175 | 155 | 124 | 75 | 35 | 8 | 4 |

Figure 7.2
The effect of a smoothing filter.
(a) Small detail of an image ( $16 \cdot 16$ pixel) shown as gray value image (left) and as number matrix (right);
(b) same as (a) after convolution with a $3 \cdot 3$ kernel (box filter, equation 7.l).

| 166 | 89 | 72 | 89 |
| :--- | :--- | :--- | :--- |
| 102 | 49 | 39 | 51 |
| 70 | 34 | 31 | 62 |
| 44 | 29 | 24 | 44 |



| 101 | 86 | 64 | 65 |
| :---: | :---: | :---: | :---: |
| 85 | 72 | 57 | 61 |
| 54 | 46 | 40 | 55 |
| 39 | 33 | 30 | 50 |

C

| 102 | 49 | 39 |
| :--- | :--- | :--- |
| 70 | 34 | 31 |
| 44 | 29 | 24 |


| 1 | $I$ | $I$ |
| :--- | :--- | :--- |
| 1 | 1 | $I$ |
| 1 | 1 | 1 |$\cdot I / 9=46$

d \begin{tabular}{|l|l|l|}
\hline 89 \& 72 \& 89 \\
\hline 49 \& 39 \& 51 \\
\hline 34 \& 31 \& 62 \\
\hline

$\cdot$

\hline I \& I \& I \\
\hline I \& I \& I \\
\hline I \& I \& I \\
\hline
\end{tabular}$\cdot \mathrm{I} / 9=57$

(a) Detail of image shown in Figure 7.2 before and after filtering, 3 pixels are highlighted;
(b) neighborhood of first pixel and calculation of average;
(c), (d) same as (b) for second and third pixel, respectively.


Figure 7.4
Box filter versus Median filter.
(a) From left to right: micrograph (500 • 375 pixel) of sheared quartzite, histogram and enlarged view of small test image ( 45.45 pixel); red frame indicates details shown in (d); small test image is shown in lower right corner at the same scale as (a);
(b) same as (a) after filtering with box filter; filter kernel is shown next to micrograph;
(c) same as (a) after Median filtering; Median filter is indicated by a row of 9 pixels sorted by gray value:
(d) effect of box filter and Median filter on small area indicated in (a); both filters are applied 10 times.


Figure 7.5
Gauss filters.
(a) Original: same as Figure 7.4.a;
(b) same as (a) after Gauss filtering; $5 \cdot 5$ filter kernel is shown next to micrograph;
(c) same as (a) after filtering; enlarged gray value representation of $15 \cdot 15$ filter kernel is shown next to test image;
(d) same as (a) after filtering; enlarged gray value representation of $45 \cdot 45$ filter kernel is shown next to test image.

The true size of the filter kernels is indicated by little squares in the lower right of the micrographs (b) to (d).


Figure 7.6
Size of filter kernel.
(a) From left to right: $3 \cdot 3,5 \cdot 5,7 \cdot 7$, and $15 \cdot \mathrm{I} 5$ neighborhood with central pixel (red);
(b) gray value representations of filter kernels: from top to bottom: $3 \cdot 3$ averaging filter kernels shown in Figures 7.7.b, 7.7.c and 7.7.d, respectively;
(c) from left to right: three Gauss filters (5 • 5, 7 7 and I5 15 ).


Figure 7.7
Influence of shape of filter. (a) Original: same as Figure 7.4.a; (b) same as (a) after filtering; box filter kernel is shown next to micrograph; (c) same as (a) after filtering; '+' shaped filter kernel is shown next to micrograph; (d) same as (a) after filtering; ' $x$ ' shaped filter kernel is shown next to micrograph; (e) same as (a) after filtering using 'Smooth' command (Process menu).

original

b


| 1 | 1 | 2 | 2 | 2 | 1 | 1 |
| :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| 1 | 2 | 2 | 4 | 2 | 2 | 1 |
| 2 | 2 | 4 | 8 | 4 | 2 | 2 |
| 2 | 4 | 8 | 16 | 8 | 4 | 2 |
| 2 | 2 | 4 | 8 | 4 | 2 | 2 |
| 1 | 2 | 2 | 4 | 2 | 2 | 1 |
| 1 | 1 | 2 | 2 | 2 | 1 | 1 |



## Gaussian 7-7

C

| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| :--- | :--- | :--- | :--- | :--- | :--- | :--- |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |
| 1 | 1 | 1 | 1 | 1 | 1 | 1 |



Box filter 7-7
d


Filter $=$ test image

Figure 7.8
Influence of size and shape of filter kernel.
(a) Original: same as Figure 7.4.a;
(b) same as (a) after Gauss filtering; $7 \cdot 7$ filter kernel is shown next to micrograph;
(c) same as (a) after filtering with box filter; $7 \cdot 7$ filter kernel is shown next to micrograph;
(d) same as (a) after filtering with $45 \cdot 45$ filter kernel: enlarged gray value representation of filter kernel (= test image) is shown next to micrograph.

## original


b

| 0 | 0 | 0 |
| :---: | :---: | :---: |
| 0 | 1 | -1 |
| 0 | 0 | 0 |

Horizontal point separation I px


C


| 0 | 0 | 0 |
| :---: | :---: | :---: |
| 1 | 0 | -1 |
| 0 | 0 | 0 |

Horizontal point separation 2 px



Horizontal line separation 2 px



| 1 | 1 | 1 |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
| -1 | -1 | -1 |

Vertical line separation 2 px


Figure 7.9
Gradient filters. (a) Original: same as Figure 7.4.a; (b) to (f) same as (a) after gradient filtering: (b) horizontal point separation, separation = I pixel; (c) horizontal point separation, separation = 2 pixel; (d) horizontal line separation, separation = 2 pixel; (e) vertical line separation, separation $=2$ pixel; $\ln (b-e)$, filter kernel is shown next to micrograph; gray values are scaled such that minimum difference $=0$ (white) and maximum difference $=255$ (black).


## C



Image SXM Shadow West


Figure 7.10
Shadows.
(a) Original: same as Figure 7.4.a;
(b) same as (a) after application of North Shadow filtering;
(c) same as (a) after application of West Shadow filtering.


## b



## C

SW-NE

| 0 | 0 | -1 |
| :---: | :---: | :---: |
| 0 | $I$ | 0 |
| 0 | 0 | 0 |

SE-NW

| 0 | -1 | 0 |
| :---: | :---: | :---: |
| 0 | 0 | 1 |
| 0 | 0 | 0 |

NE-SW

| 0 | 0 | 1 |
| :---: | :---: | :---: |
| 0 | -1 | 0 |
| 0 | 0 | 0 |

NW-SE

| 0 | I | 0 |
| :---: | :---: | :---: |
| 0 | 0 | -I |
| 0 | 0 | 0 |



## e


f


Figure 7.11
Roberts cross.
(a) Original: same as Figure 7.4.a;
(b) same as (a) after application of SW-NE step;
(c) 4 gradient filters of Roberts Cross;
(d) test image after application of filters shown in (c);
(e) maximum of 4 images shown in (d), contrast enhanced;
(f) thresholded version of (e).


Figure 7.12
Edge detection with Roberts cross.
(a) Original: same as in Figure 7.4.a, left;
(b) image after application of filters shown in Figure 7.I I.c;
(c) maximum of 4 images shown in (b), contrast enhanced;
(d) thresholded version of (c).

| 1 | 2 | 1 |
| :---: | :---: | :---: |
| 0 | 0 | 0 |
| -1 | -2 | -1 |


| 1 | 0 | -1 | -I | -2 | -1 |
| :---: | :---: | :---: | :---: | :---: | :---: |
| 2 | 0 | -2 | 0 | 0 | 0 |
| 1 | 0 | -I | 1 | 2 | 1 |


| -1 | 0 | 1 |
| :---: | :--- | :--- |
| -2 | 0 | 2 |
| $-I$ | 0 | 1 |


| -2 | -1 | 0 |
| :---: | :---: | :---: |
| -1 | 0 | 1 |
| 0 | 1 | 2 |


| 0 | $I$ | 2 |
| :---: | :---: | :---: |
| $-I$ | 0 | $I$ |
| -2 | $-I$ | 0 |


| 2 | 1 | 0 |
| :---: | :---: | :---: |
| 1 | 0 | -1 |
| 0 | -1 | -2 |


| 0 | -1 | -2 |
| :---: | :---: | :---: |
| 1 | 0 | -1 |
| 2 | 1 | 0 |

b



Figure 7.13
Sobel operator.
(a) 8 gradient filters;
(b) test image (c) after application of filters shown in (a);
(c) original test image;
(d) maximum of 8 images shown in (c), contrast enhanced;
(e) thresholded version of (d).


Figure 7.14
Edge detection with Sobel operator.
(a) Image (b) after application of filters shown in Figure 7.I3.a;
(b) original image;
(c) maximum of 8 images shown in (c), contrast enhanced;
(d) thresholded version of (c).

## a



## C




## d



Figure 7.15
Edge detection in Image SXM.
(a) Find Edges (Sobel edge detection);
(b) Find Edges ( $5 \times 5$ );
(c) 'hand made' Sobel edge detection (as in Figure 7.14.c);
(d) Roberts Cross, for comparison.


| -1 | -1 | -1 | -1 | -1 |
| :--- | :--- | :--- | :--- | :--- |
| -1 | -1 | -1 | -1 | -1 |
| -1 | -1 | 40 | -1 | -1 |
| -1 | -1 | -1 | -1 | -1 |
| -1 | -1 | -1 | -1 | -1 |

Laplace 5-5


| -1 | -1 | -1 |
| :--- | :--- | :--- |
| -1 | 12 | -1 |
| -1 | -1 | -1 |

Laplace 3 • 3

Laplace 3 • 3


Figure 7.16
Laplace filter for sharpening.
(a) Original: blurred version of Figure 7.4.a;
(b) same as (a) after application of $5 \cdot 5$ Laplace filter;
(c) same as (a) after application of $3 \cdot 3$ Laplace filter;
(d) same as (a) after applying 3•3 Laplace filter two times; filter kernels are shown next to micrographs.

b


| -1 | -1 | -1 |
| :---: | :---: | :---: |
| -1 | 8 | -1 |
| -1 | -1 | -1 |

Laplace 3 - 3


C


| -1 | -1 | -1 |
| :---: | :---: | :---: |
| -1 | 8 | -1 |
| -1 | -1 | -1 |

Laplace 3 - 3


Figure 7.17
Edge detection with Laplace filter.
(a) Original: similar to Figure 7.4.a;
(b) same as (a) after application of $3 \cdot 3$ Laplace filter;
(c) same as (b) after thresholding.

## a






C



Figure 7.18
Effect of Gauss filter on gray value profiles.
(a) Original image (same as Figure 7.4.a) and gray value profile along trace;
(b) image (a) after $5 \cdot 5$ Gauss filtering;
(c) difference image obtained by subtracting (b) from (a).
import raw image [0]
crop and scale - fast [1]
crop and scale - smooth [2]
Median filter stack [U]
Sobel filter stack [O]
sharpen stack [H]
enhance stack [C]
equalize stack [E]
threshold stack [B]
adaptive -mean- threshold stack [G]
density slice stack [D]
skeletonize stack [K]
prune stack $[\mathrm{P}]$
average of stack [A]
max of stack [Z]
max of windows into first [W]
Median filter image [ M ]
prune image [I]
skeletonize image [J]
equalize image [Q]
density slice image about 128 [L]
thicken lines [T]
smooth image [S]
smooth image more [U]

## cut away rim [4]

black rim for boundary map [5]
white rim for grain map [6]
smooth enlarge of outlines [8]
invert image [ Y ]
scale to pixel $[X]$
info on histo [F]
black area percent of bitmap [\$]
(I) file opening and re-sizing
(2) working with the stack
(3) combining the slices
(4) working with the combined image
(5) editing the grain boundary map



C



Figure 7.20
Lazy grain boundaries macro: working with the stack.
(a) Original image: color micrograph of naturally deformed quartzite;
(b) color channels: $\mathrm{R}=$ Red, $\mathrm{G}=\mathrm{Green}, \mathrm{B}=$ Blue; arrows point to same site in $\mathrm{R}, \mathrm{G}$, an B ;
(c) same as (b), after application of histogram equalization;
(d) same as (c) after Median filtering (2 times);
(e) same as (d) after Sobel edge detection; arrows point to same site;
(f) same as (e) after thresholding;
(g) same as (f) after Median filtering;
(h) same as (g) after skeletonizing; arrows point to same site;
(i) original with sum of (h) superposed.


(1)
(2)

Figure $\mathbf{7 . 2 0}$



Figure 7.21
Lazy grain boundaries macro: working with the combined image.
Slices of segmented stack are combined at various points of post-processing:
(a) after Sobel filtering [O] (Figure 7.20.e);
(b) after thresholding [G] (Figure 7.20.f);
(c) after median filtering [U] (Figure 7.20.g);
(d) after skeletonization [J] (Figure 7.20.h).


Figure 7.22
Lazy grain boundaries macro: creating grain boundary maps.
Further processing using image shown in Figure 7.2I.b (uog-z) as starting point, the key strokes of the applied macros (Figure 7.19) are indicated:
(a) after median filtering (3 times);
(b) same as (a) after skeletonizing;
(c) same as (b) after pruning, line thickening, skeletonizing and pruning;
(d) same as (c) after thickening of lines and blackening the grains that cut the image boundary.

