

Figure 9.1
Geometric attributes of an ellipse.
(a) Ellipse: $\mathrm{a}=$ long axis, $\mathrm{b}=$ short axis; $\mathrm{b} / \mathrm{a}=$ axial ratio; $\alpha_{i}=$ angle with respect to x -axis; $\mathrm{X}_{\mathrm{c}} \mathrm{Y}_{\mathrm{c}}=\mathrm{x}-\mathrm{y}$ coordinates of center point;
(b) Exponent n yields exact shape of ellipse.


C

|  | x |
| ---: | ---: |
| 1.000 | 0.000 |
| 0.985 | 0.087 |
| 0.940 | 0.171 |
| 0.866 | 0.250 |
| 0.766 | 0.321 |
| 0.643 | 0.383 |
| 0.500 | 0.433 |
| 0.342 | 0.470 |
| 0.174 | 0.492 |
| 0.000 | 0.500 |
| -0.174 | 0.492 |
| -0.342 | 0.470 |
| -0.500 | 0.433 |
| -0.643 | 0.383 |
| -0.766 | 0.321 |
| -0.866 | 0.250 |
| -0.940 | 0.171 |
| -0.985 | 0.087 |
| -1.000 | 0.000 |
| -0.985 | -0.087 |
| -0.940 | -0.171 |
| -0.866 | -0.250 |
| -0.766 | -0.321 |
| -0.643 | -0.383 |
| -0.500 | -0.433 |
| -0.342 | -0.470 |
| -0.174 | -0.492 |
| 0.000 | -0.500 |
| 0.174 | -0.492 |
| 0.342 | -0.470 |
| 0.500 | -0.433 |
| 0.643 | -0.383 |
| 0.766 | -0.321 |
| 0.866 | -0.250 |
| 0.940 | -0.171 |
| 0.985 | -0.087 |
| 1.000 | 0.000 |
| 99999 | 999999 |

Figure 9.2
Parameter description of an ellipse.
(a) Ellipse: $x, y=$ coordinates on outline, $\varphi=$ angle with respect to $x$-axis;
(b) inscribed polygon connecting $36 x-y$ coordinates on elliptic outline;
(c) list of $x-y$ coordinates plotted in (b), last coordinate $=$ end coordinate (= separator coordinate) is not plotted.


Figure 9.3
Three distribution functions for $(0<x<1)$.
(a) Uniform distribution: $\mathrm{h}(\mathrm{x})=1.00$ for for ( $0 \leq x \leq 1.00$ );
(b) Gaussian normal distribution: $h(x)=\mu=0.5$ and $\sigma=0.1465$;
(c) monodisperse distribution (= delta function): $h(x)=\infty$ if $x=0.5, h(x)=0.00$ if $x \neq 0.5$.

| fabric <br> feature | distribution |  |  |
| :---: | :--- | :--- | :--- |
|  | monodisperse | normal | uniform |
| a | perfectly sorted <br> all the same size | well sorted: <br> small $\sigma$ <br> poorly sorted: <br> large $\sigma$ | all grain sizes are <br> equally likely |
| b/a | identical axial ratio <br> for all ellipses | one predominant <br> axial ratio | everything from <br> line to circle |
| $\alpha$ i | parallel ellipses | preferred <br> orientation, <br> subparallel ellipses | random <br> orientation |

## Table 9.1

Effect of different types of distribution functions:
a = long axis;
b/a = short axis / long axis = axial ratio;
$\alpha_{i}=$ angle of orientation of long axis.


b

C

Figure 9.4
Orientation distribution function (ODF) for non-polar directions in the range of $\left(0^{\circ}<\alpha_{i}<180^{\circ}\right)$.
(a) Uniform ODF: random orientation;
(b) Normal ODF: preferred orientation;
(c) Monodisperse ODF: parallel objects (ellipses, lines);

Left: shown as distribution function; center: shown as rose diagram; right: corresponding line fabric.


Figure 9.5
Spatial distribution of ellipses.
All ellipses are of the same size; they all have the same axial ratio (b/a $=0.7$ ); and all are parallel to the $x$-axis $\left(\alpha_{i}=0^{\circ}\right)$.
(a) Arranged on regular grid;
(b) perfectly random distribution (Poisson distribution) of center points (not realized in nature);
(c) isotropic distribution of center points: minimum distance between center points is larger than long diameter of ellipses;
(d) anisotropic distribution of center points: ratio of vertical to horizontal minimum distance is 0.7 (corresponding to b/a of ellipse).


Figure 9.6
Constant fabric features (delta functions).
Size, a, axial ratio, b/a, and orientation, $\mathrm{a}_{\mathrm{i}}$, of particles, exponent, n , and number of points per particle, N , are constant (ie., monodisperse):
(a) $\mathrm{b} / \mathrm{a}=0.00, \mathrm{a}=0.7, \mathrm{a}_{\mathrm{i}}=0^{\circ}, \mathrm{N}=2$;
(b) $\mathrm{b} / \mathrm{a}=0.25, \mathrm{a}=0.7, \mathrm{a}_{\mathrm{i}}=135^{\circ}, \mathrm{N}=36, \mathrm{n}=\mathrm{I}$;
(c) $\mathrm{b} / \mathrm{a}=1.00, \mathrm{a}=0.7, \mathrm{a}_{\mathrm{i}}=0^{\circ}, \mathrm{N}=36, \mathrm{n}=\mathrm{I}$;
(d) $\mathrm{b} / \mathrm{a}=0.50, \mathrm{a}=0.7, \mathrm{a}_{\mathrm{i}}=60^{\circ}, \mathrm{N}=36, \mathrm{n}=0.5$;
(e) $\mathrm{b} / \mathrm{a}=0.50, \mathrm{a}=0.7, \mathrm{a}_{\mathrm{i}}=90^{\circ}, \mathrm{N}=4, \mathrm{n}=2$.


Figure 9.7
Normal distribution of fabric features
One or more of the fabric features follow the Gaussian normal distribution:
(a) normal distribution of size: $\mu(a)=0.7, \sigma(a)=0.15$;
(b) normal distribution of axial ratio: $\mu(b / a)=0.7, \sigma(b / a)=0.1$;
(c) normal distribution of orientation: $\mu\left(\alpha_{i}\right)=0^{\circ}, \sigma\left(\alpha_{i}\right)=15^{\circ}$;
(d) normal distribution of size as in (a) and axial ratio as in (b);
(e) normal distribution of axial ratio as in (b) and orientation as in (c);
(f) normal distribution of size as in (a), axial ratio as in (b) and orientation as in (c).


Figure 9.8
Random fabric features
One or more of the fabric features are uniformly distributed:
(a) uniform distribution of size, a, (random size);
(b) uniform distribution of axial ratio, b/a, (random axial ratio);
(c) uniform distribution of orientation, $\alpha_{i}$, (random orientation);
(d) uniform distribution of size, a, and axial ratio, b/a;
(e) uniform distribution of axial ratio, b/a, and orientation, $\alpha_{i}$;
(f) uniform distribution of size, a, axial ratio, $\mathrm{b} / \mathrm{a}$, and orientation, $\mathrm{a}_{\mathrm{i}}$.

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RANDOM generates synthetic fabrics
strings of x-y coordinates are saved in formatted file
separator coordinate is 99999
max. no. of ellipses = 1000 max. no. of pts. = 370
```

no. of ellipses, no. of pts./ellipse >
I 100,36
no. of particles for scaling (default = 100) ? >
4 start numbers ( 1 to 10 ) for random number generator input four integers ("a,b,c,d") >
$3 \quad 1,2,3,4$
specify the spatial distribution of the particles: random position (1) or position on grid (0) >
1
min.distance between center points (w/r to size) (possible values: 0.-1.00) >
$5 \quad .75$
isotropic or anisotropic positioning ? iso=1, aniso=0 >
1
type of orientation distribution (ODF) of long axes ?
1: ODF=uniform (random), 0: ODF=normal (pref. or.),
-1: ODF=delta function (parallel), 2: manual input
mean and standard deviation (degrees) >
$8 \quad 0 ., 10$.
type of distribution function (h(a)) of long axes
1: h(a)=uniform (all sizes), 0: h(a)=normal (pref. size),
-1: h(a)=delta function (only one size) >
length of long axes in rel.units (values: 0.-1.00) >
1: h(b/a)=uniform (all), 0: h(b/a)=normal (pref.ratio),
-1: h(b/a)=delta function (only one ratio) >

1
exact shape ? (0.= rectangle, 1.= ellipse, 2.= rhomb)
name of file >
data
header (maximum length $=132$ characters) >
14100 ellipse, $\mathrm{N}=36, \mathrm{n}=1$, $\mathrm{alf}=0 \pm 10$, $\mathrm{a}=.7$, $\mathrm{b} / \mathrm{a}=$ random
file will be saved as formatted file;
= input for surfor, paror, shapes, etc.

Software Box 9.1
Dialog with program RANDOM.
subroutine chance (xmean, sigma, r)
calculates normally distributed pseudorandom numbers
xmean real input mean of distribution
sigma real input standard deviation
r
real output
pseudorandom number
rsum $=0$.
do 70 i $=1,12$
rz=rand(0)
rsum $=$ rsum $+r z$
$r=$ (rsum - 6.) * sigma + xmean
return
end

(a) Gaussian normal distribution (light gray background = uniform distribution);
(b) lognormal distribution;
(c) asymmetric distribution (log-sine);
(d) bimodal distribution.


## Figure 9.10

Random sectioning of spheres with distributed size of radius $R$
Histograms show random samples of spheres, $R$ (black bars), subsamples of $R$ that have been hit by the sectioning plane (gray bars) and resulting samples of sectional circles, $s$ (white bars). Sample size, N , of sections is always 100 .
(a) $h(R)$ is the standard uniform distribution; interval $(0, I)$; total number of spheres, $N_{R}=204$;
(b) $h(R)$ is the Gaussian normal distribution; mean $=0.5$, standard deviation $=0.1$; total number of spheres, $N_{R}=200$;
(c) $h(R)$ is the bimodal distribution; total number of spheres, $N_{R}=214$;
(d) corresponding schematic drawing of sphere (black), sphere hit by sectioning plane (grey) and resulting sectional circle (white).


Figure 9.1 I
Sectional circles of spheres with distributed size of radius $R$
100 sections of 3 different distributions of spheres have are shown (see histograms of Figure 9.10). For comparison, 100 sections of spheres with identical radius are shown also (d).
(a) $h(R)$ is the standard uniform distribution;
(b) $h(R)$ is the Gaussian normal distribution;
(c) $h(R)$ is the bimodal distribution;
(d) $h(R)$ is the monodisperse distribution.

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this is MONTECARLO
24-aug-2010
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the program creates random samples of section ,s
obtained by sectioning populations of spheres, R
the size distribution of the spheres must be specified
$1-$ standard uniform $h(R)=1.00(0.00 \leq R \leq 1.00)$
2- Gaussian normal or 3- General distribution $h(R)$
for 3 need file with 101 values of $h(R)$ (interval dR=0.01)
$(0.00 \leq h(R) \leq 1.00)$ and $(0.00 \leq R \leq 1.00)$,
the output file contains a list of $N$ values of $R$ and $s$
\$no.of sections (size N of sample) >
\$seeds for random distance and radius (2 integers) >
2,3
\$distribution ? 1-uniform 2-normal 3-other (1 integer) >
\$name of distribution file >
bil01.dst
\$name of file for $R$ (spheres) and $s$ (sections >
bil00.txt

