



Figure 10.1
Micrograph and bitmaps of granitoid rock.
(a) Original image, SEM micrograph, BSE contrast;
(b) histograms of original (a) with limits between phases indicated, phase map (c) and grain map (d);
(c) phase map of (a);
(d) grain map of (a);
see Table IO.I for gray values of minerals.

| gray value <br> in phase map | color | mineral | abbreviation |
| :---: | :---: | :---: | :---: |
| 10 | white | biotite | bio |
| 63 | very light grey | K- feldspar ${ }^{\text {I }}$ | kfs |
| 87 | light grey | muscovite | mu |
| 150 | medium grey | plagioclase | plg |
| 181 | dark grey | quartz | qtz |
| 250 | nearly black | unidentified ${ }^{2)}$ |  |
| 255 | black | grain boundaries ${ }^{3)}$ | gb |

## Table $\mathbf{1 0 . 1}$

Gray values of five mineral phases in segmented image (Figure IO.I).
I) orthoclase; 2) mostly holes; includes grain and phase boundaries.


| Info |  |
| :---: | :---: |
| $x:$ | 96 |
| Y: | 322 |
| YaI: | 0 |
| Count | \# 1 |
| Area: | 103917 sq pix |
| Hean: | 173.339 (68\%) |
| StDv: | 8.502 |
| Hin: | 158.000 |
| Hax: | 200.000 |
| Phx: | 29.676 \% |
| Lox: |  |
| Hix: | 1.850 各 |

$q t z=29.7 \%$
$q t z=28.8 \%$
$q t z=26.8 \%$

## Figure $\mathbf{1 0 . 2}$

Measuring Density Slices.
Using the LUT tool, density slices are created for:
(a) the original SEM micrograph;
(b) the phase map;
(c) the grain map;
the range of gray values is set for quartz in all cases.


Figure 10.3
Measuring while Thresholding.
(a) Threshold (at GV=|58) applied to original SEM micrograph; qtz+ indicates that quartz phase includes holes and grain boundaries;
(b) threshold ( $\mathrm{GV}=\mathrm{I} 28$ ) applied to bitmap of quartz phase;
(c) threshold ( $\mathrm{GV}=128$ ) applied to bitmap of quartz grains.


## Table I 0.2

Comparison of area evaluations.
6 phases $=5$ mineral phases +1 unidentified phase (holes, etc.);
7 phases $=5$ mineral phases +1 unidentified phase (holes, etc.) + grain boundary 'phase';
5 phases $=5$ mineral phases only.


## Figure $\mathbf{1 0 . 4}$

Determining area fractions.
Detail of a synthetic calcite (dark) - anhydrite (light) mixture is shown as phase map (top row) or as grain map with grain boundaries (bottom row).
(a) Using point fractions of a counting grid;
(b) using line fractions on test lines;
(c) counting the pixel on each of the phases.

b

C

$\mathrm{A}_{\mathrm{tot}}$

Figure 10.5
Volume fractions from area fractions.
(a) Schematic of a volume, $V_{\text {tot }}$, containing the phase $\alpha$; the position of a section (thickness $=\Delta z$, volume $=\Delta V$ ) is indicated;
(b) the $x-y$ section shown in plane view; the total area of the image is $A_{\text {tot }}$; the fractional area of phase, $A_{\alpha}$, is the sum of all the cross sectional areas of $\alpha$;
(c) plot of $A_{\alpha}$ as a function of $z$; the total image area, $A_{\text {tot }}$, is constant; the fractional area, $A_{\alpha}$, changes; the average area of $\alpha, \bar{A}_{\alpha}$, is indicated; the black bar marks the position of the section shown in (b).

## a


relative error $=I / \sqrt{ } N_{\text {tot }}$
$\mathrm{I} / \sqrt{ } \mathrm{N}_{\text {tot }}=\mathrm{I} / \sqrt{ } 167=7.7 \%$
$\Rightarrow$ quartz content: $29.4 \% \pm 2.3 \%$

relative error $\left.=\sqrt{ }\left((\sigma(a) / \bar{a})^{2}+1\right) / N_{\text {tot }}\right)$

| $\bar{a}$ | $=1924$ square pixel |
| :--- | :--- |
| $\sigma(a)$ | $=2586$ square pixel |
| $\sigma(\mathrm{a}) / \overline{\mathrm{a}}$ | $=1.344$ |
| $(\sigma(\mathrm{a}) / \overline{\mathrm{a}}))^{2}+1$ | $=\left(1.344^{2}+1\right)=2.806$ |

$$
\left.\sqrt{ }\left((\sigma(\mathrm{a}) / \overline{\mathrm{a}})^{2}+\mathrm{I}\right) / \mathrm{N}\right)=\sqrt{ }(2.806 / 166)=13.0 \%
$$

(incl. particles touching edges)

$$
\Rightarrow \text { quartz content }=29.4 \pm 3.8 \%
$$

## Figure $\mathbf{1} 0.6$

Error estimates from the total number of grains.
Quartz content and error of estimate are calculated
(a) considering the total number of grains, N , only;
(b) considering the total number of grains, N , and the relative size variation, $\sigma(\mathrm{a}) / \overline{\mathrm{a}}$, of the cross sectional areas of all grains.

relative error $=1 / \sqrt{ } \mathrm{N}_{\alpha}$

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I/\sqrt{}{N}N=I/\sqrt{}{}(47)=I4.6%
=> quartz content: 29.4 % \pm4.3 %
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relative error $\left.=\sqrt{ }\left((\sigma(a) / \bar{a})^{2}+I\right) / N_{\alpha}\right)$
$\overline{\mathrm{a}} \quad=1996$ square pixel
$\sigma(a) \quad=1999$ square pixel
$\sigma(\mathrm{a}) / \overline{\mathrm{a}}=1.002$
$(\sigma(\mathrm{a}) / \overline{\mathrm{a}}))^{2}+1 \quad=\quad\left(1.002^{2}+1\right)=2.003$
$\left.\sqrt{ }\left((\sigma(\mathrm{a}) / \overline{\mathrm{a}})^{2}+\mathrm{I}\right) / \mathrm{N}\right)=\sqrt{ }(2.003 / 47)=20.6 \%$
(incl. particles touching edges)

$$
\Rightarrow \text { quartz content }=29.4 \% \pm 6.1 \%
$$

## Figure $\mathbf{1 0 . 7}$

Error estimates from the number of quartz grains.
Quartz content and error of estimate are calculated
(a) considering the number of quartz grains, $\mathrm{N}_{\alpha}$, only;
(b) considering the number of quartz grains, $N_{\alpha}$, and the relative size variation, $\sigma(\mathrm{a}) / \overline{\mathrm{a}}$, of the cross sectional areas of the quartz grains.

| phase | area (\%) | $\#$ | relative error <br> $I / \sqrt{ } N$ | 1 <br> $\pm(\%)$ | 2 <br> $\pm(\%)$ | 3 <br> $\pm(\%)$ | 4 <br> $\pm(\%)$ |
| :--- | :---: | :---: | :---: | :---: | :---: | :---: | :---: |
| biotite | 7.25 | 15 | 0.258 | 0.56 | 0.94 | 1.87 | 4.00 |
| K- feldspar | 20.73 | 32 | 0.177 | 1.60 | 2.70 | 3.67 | 5.82 |
| muscovite | 2.23 | 7 | 0.378 | 0.17 | 0.29 | 0.84 | 1.21 |
| plagioclase | 40.41 | 66 | 0.123 | 3.13 | 5.25 | 4.97 | 9.11 |
| quartz | 29.38 | 47 | 0.146 | 2.27 | 3.82 | 4.29 | 6.06 |
|  |  |  |  |  |  |  |  |
| all | 100.00 | 167 | 0.0774 | 7.74 | 13.00 | 7.74 | 12.96 |

I using relative error and $N=N_{\text {tot }} \quad 3$ using relative error and $N=N_{\text {phase }}$
2 using area variation and $N=N_{\text {tot }} 4$ using area variation and $N=N_{\text {phase }}$

## Table 10.3

Estimates and relative errors (in \%) of mineral composition of granitoid rock.
I using relative error and $\mathrm{N}=\mathrm{N}_{\text {tot }}$
2 using area variation and $N=N_{\text {tot }}$
3 using relative error and $N=N_{\text {phase }}$
4 using area variation and $N=N_{\text {phase }}$


## Figure $\mathbf{1 0 . 8}$

Determination of rock type from modal composition.
(a) Histogram showing the calculated volume fractions for 5 mineral phases, the ratio ( K -feldspar : plagioclase : quartz) is indicated below;
(b) QAP diagram with composition of analyzed sample: granodiorite.


Figure $\mathbf{1 0 . 9}$
Determination of rock type from modal composition.
(a) Histogram showing the calculated volume fractions (black) and errors (red) for 5 mineral phases, the number for Kfeldspar, plagioclase and quartz are indicated below;
(b) QAP diagram with composition of analyzed sample, including a circle outlining the maximum error (9.1 \%): granodiorite or monzogranite.



## Figure I0.II

Horizontal and vertical fractions of boundaries.
From left to right, the complete boundaries, the horizontal and the vertical fractions are shown.
(a) Grain boundaries including phase boundaries;
(b) phase boundaries only;
(c) grain boundaries, sensu strictu;
the number of pixels of the boundary 'phase' is indicated for each.

