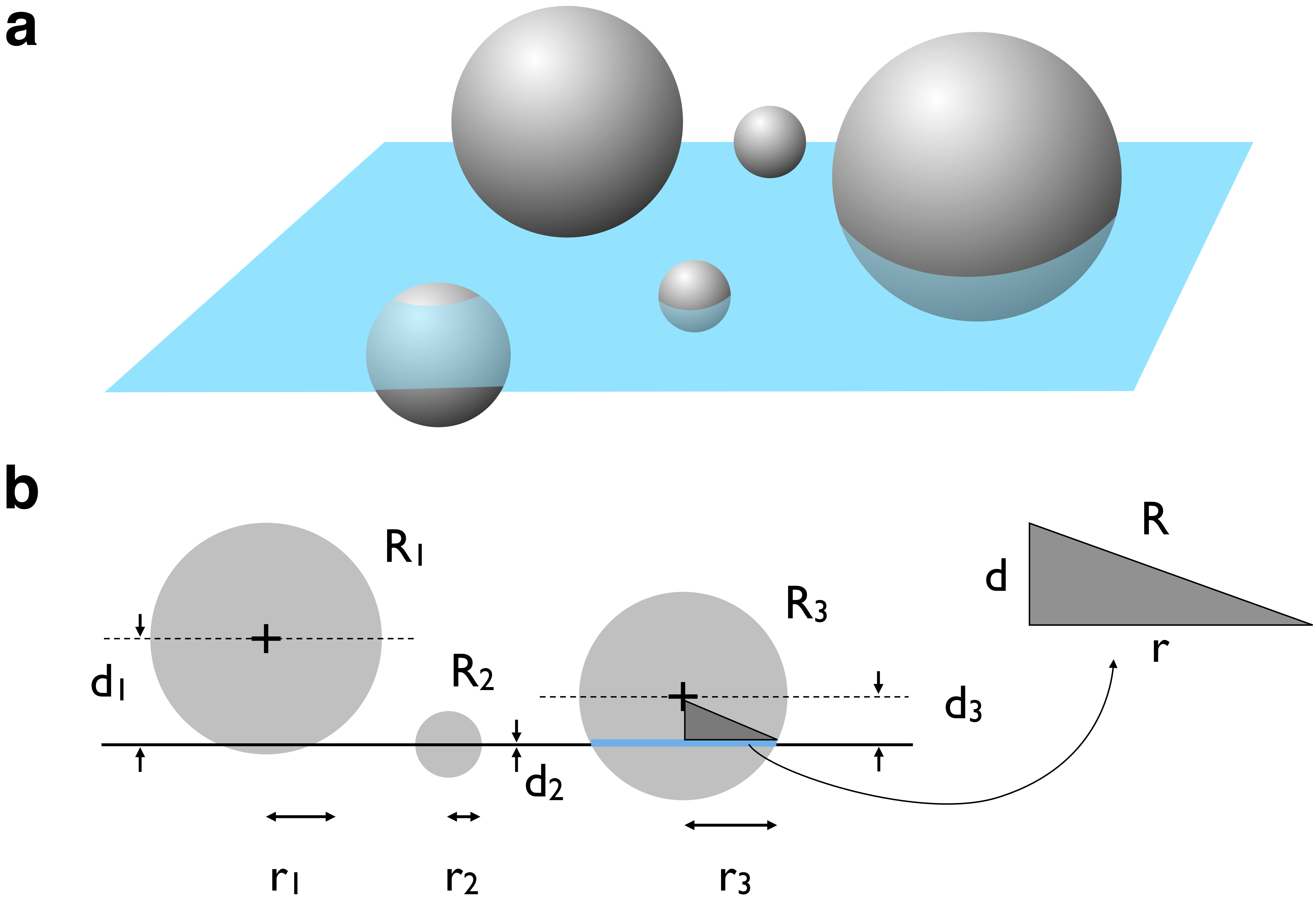

Figure 12.1
The stereological model for random sectioning.
Three spheres are sectionend.
(a) 3-D spheres and sectioning plane (light blue);
(b) derivation of radius of 2-D circle:
R = radius of sphere;
r = radius of sectional circle;
d = distance of center of sphere from sectioning plane.

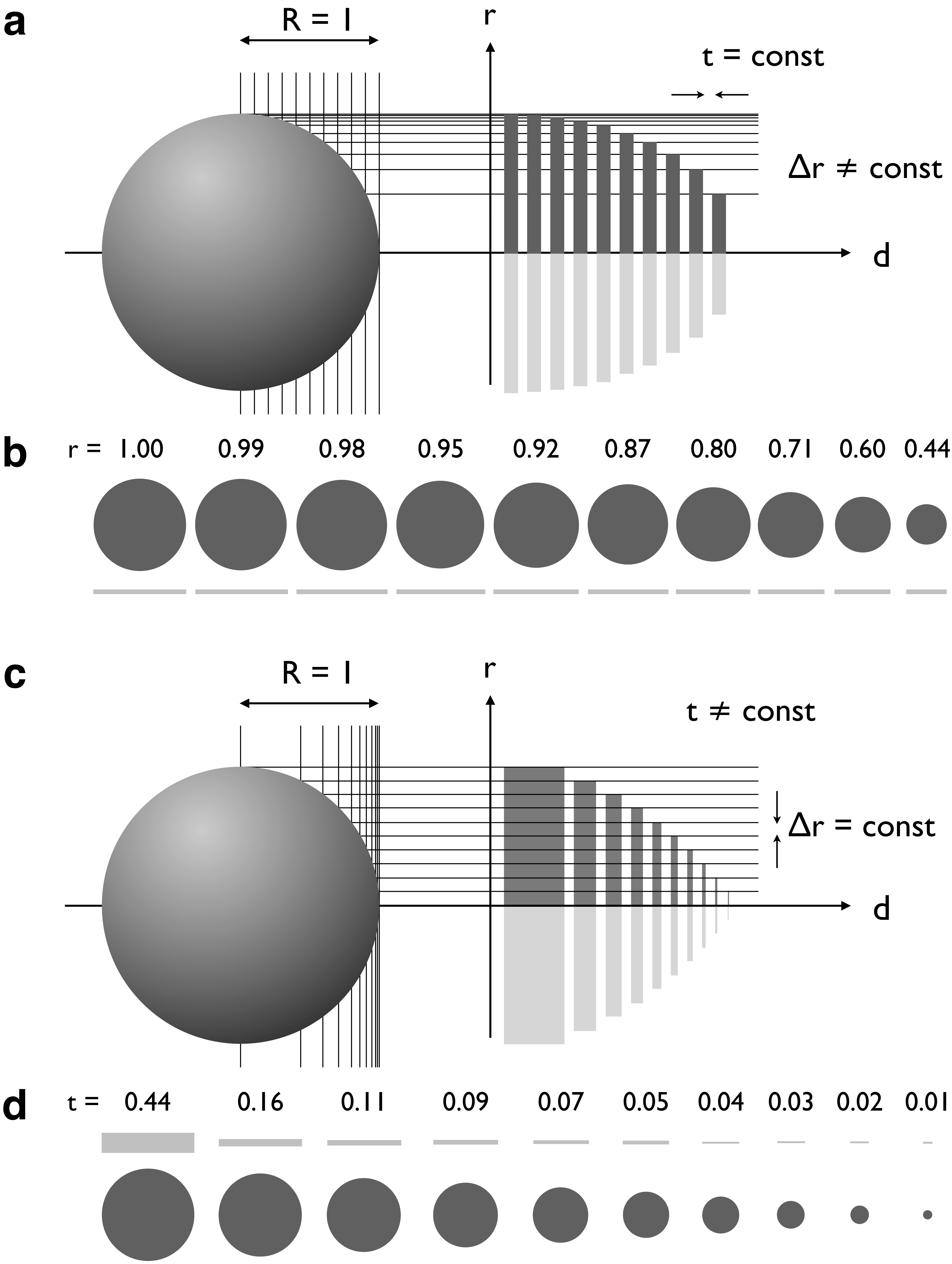

Figure 12.2
Slicing a sphere.
(a) Unit sphere with 10 sectioning planes such that $\Delta d = 0.1$ = constant;
(b) 10 slices (dark = top view; light = side view), thickness, $t = 0.1$, radius, r, indicated in fractions of R;
(c) unit sphere with 10 sectioning planes such that $\Delta r = 0.1$ = constant;
(d) 10 slices (dark = top view; light = side view), radius = 1.0, 0.9, ... 0.1, thickness, t, indicated in fractions of R.

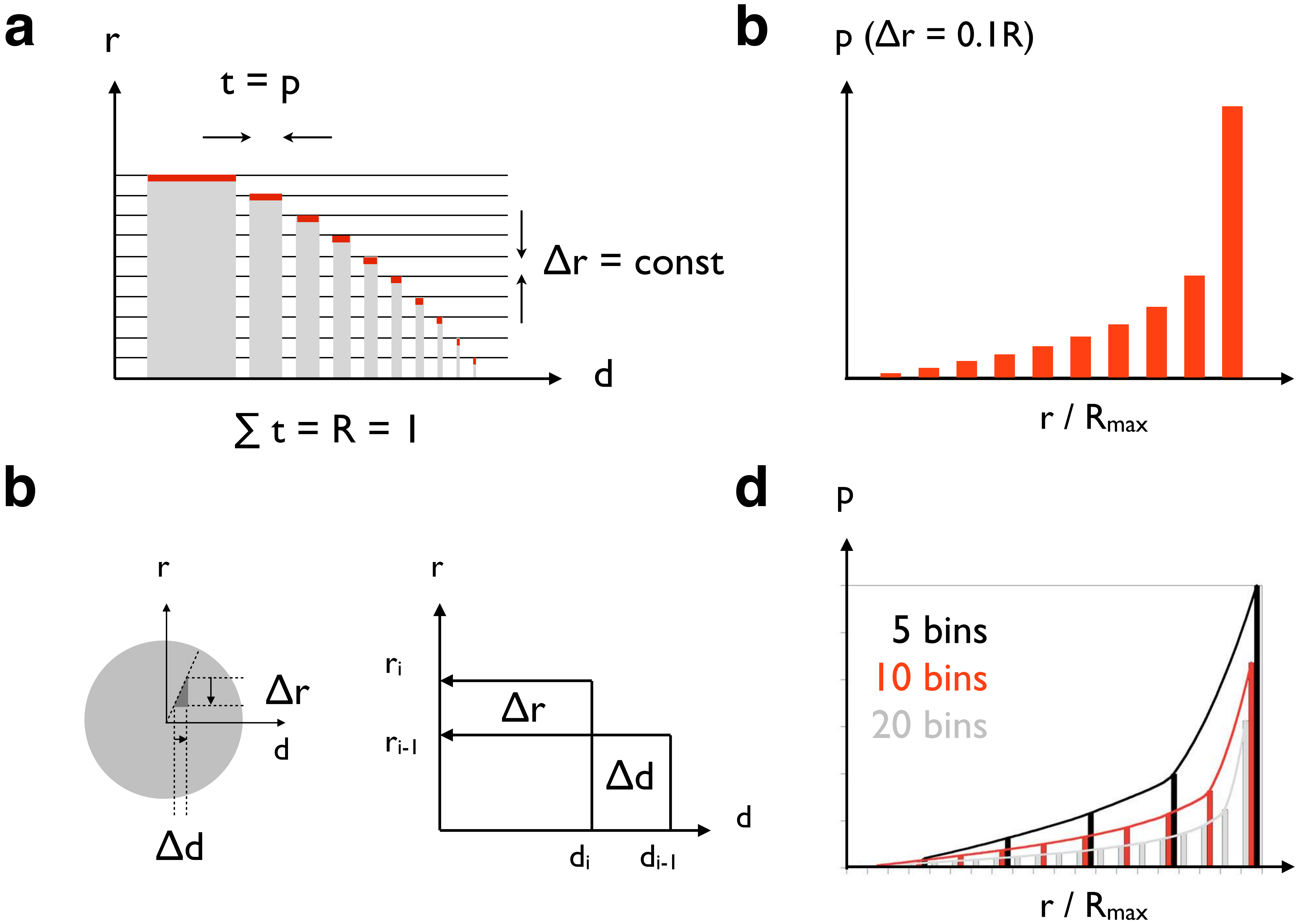

Figure 12.3
Probabilities.
(a) Unit sphere (R=1) divided into 10 slices; thickness, t, of slices is variable; increment of radius, Δr, is constant (compare Figure 12.2.c); probability to obtain slice with radius, r, is proportional to thickness of slice, t;
(b) probability, p, to obtain slice with radius, r, for increment $\Delta r = 0.1 \cdot R$;
(c) defining lower and upper limit of intervals Δr and Δd;
(d) same probability, p, as in (b), plotted for 5 bins (black), 10 bins (red) and 20 bins (gray).

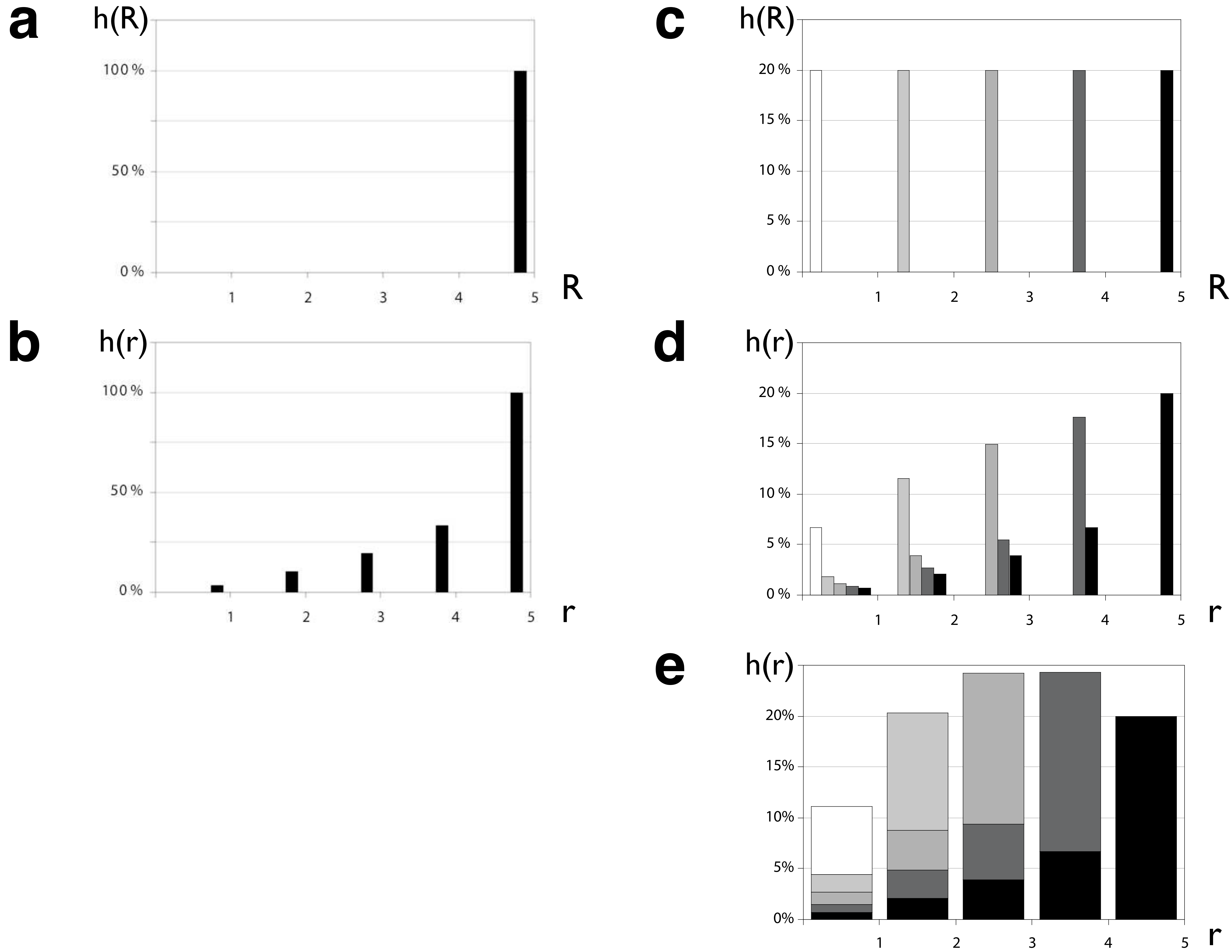

Figure 12.4
Size distribution, h(r), of sectional circles from size distribution, h(R), of spheres.
(a) h(R) = monodisperse distribution;
(b) h(r) of monodisperse h(R);
(c) h(R) = uniform distribution;
(d) h(r) of uniform h(R) with h(r) of each size class of h(R) shown separately (color-coded);
(e) h(r) of uniform h(R) with h(r) shown as the (color-coded) sum of contributions of each size class of h(R).

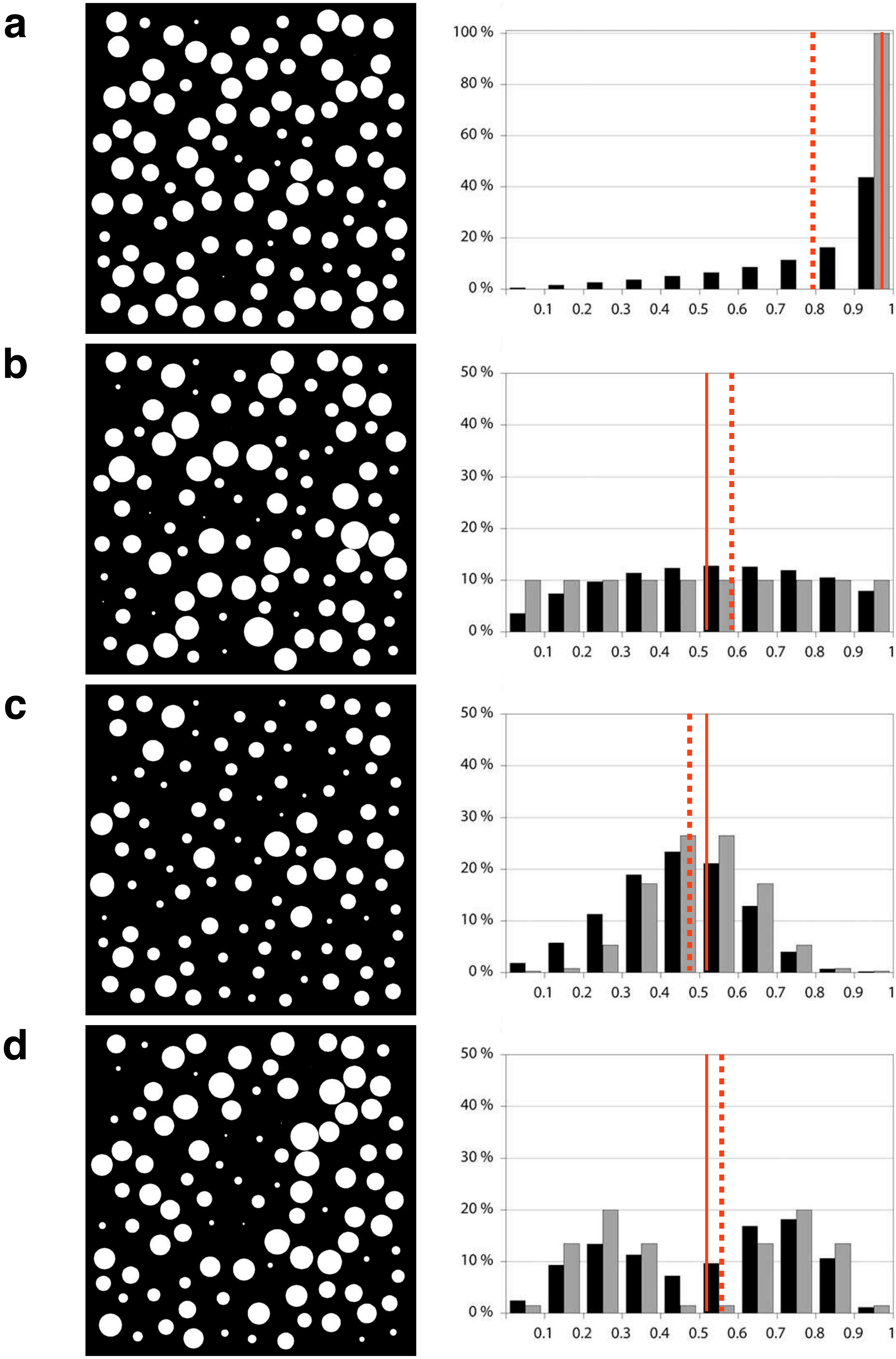

Figure 12.5
Size distribution of sectional circles for different size distribution of spheres.
Distributions, h(r), of circles calculated for: (a) h(R) = monodisperse distribution; (b) h(R) = uniform distribution; (c) h(R) = Gaussian normal distribution; (d) h(R) = bimodal distribution; left: bitmap of distribution h(r); right: histograms, h(R) (gray) and h(r) (black); stippled red lines = means of h(r), solid lines = means of h(R).

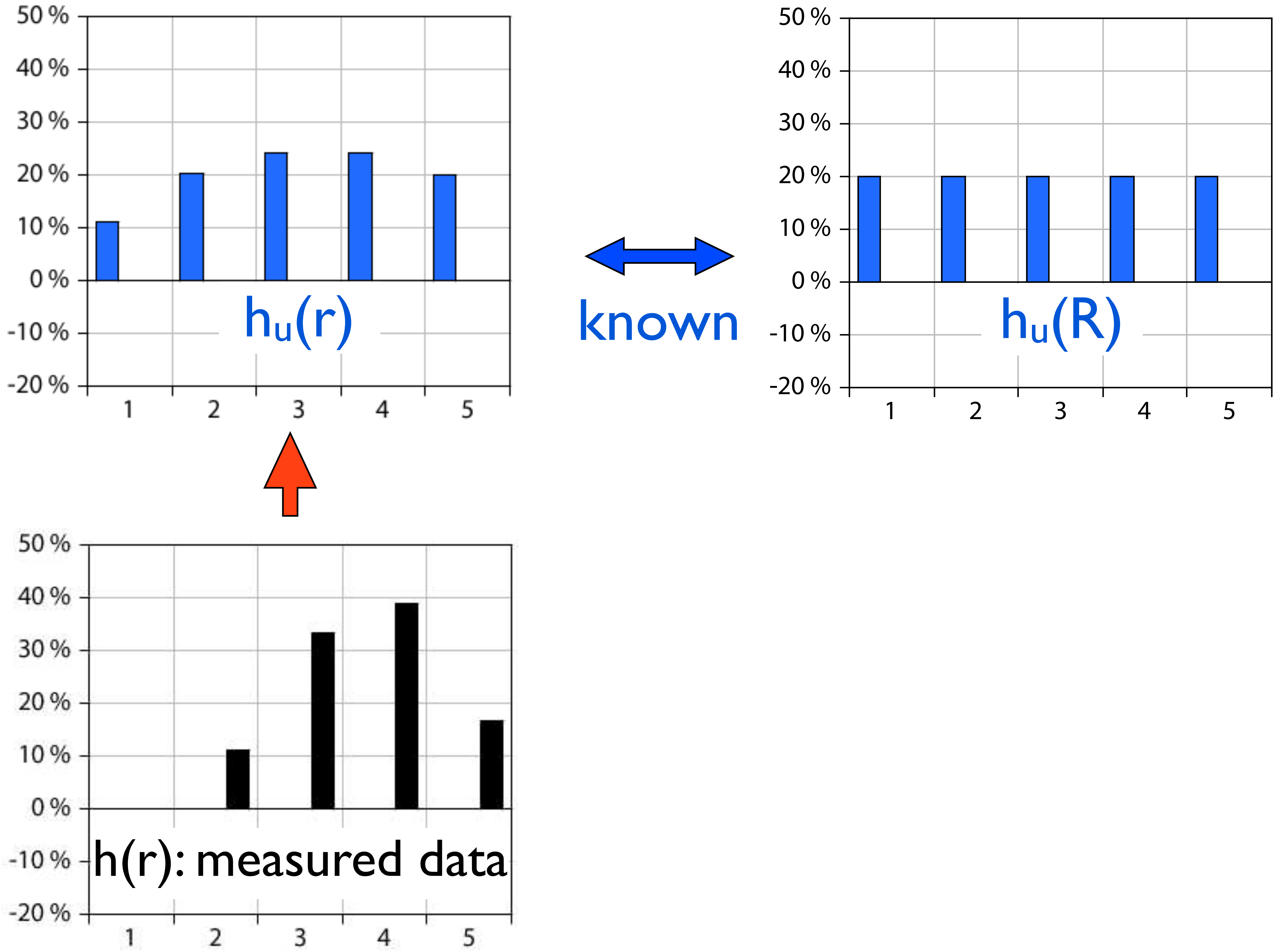

Figure 12.6
Basic idea behind the STRIPSTAR program.
For any uniform size distribution of spheres, $h_u(R)$, the size distribution of sections, $h_u(r)$ can be calculated. By comparing a measured size distribution, $h(r)$, with $h_u(r)$, the parent distribution of spheres, $h(R)$, can be derived.

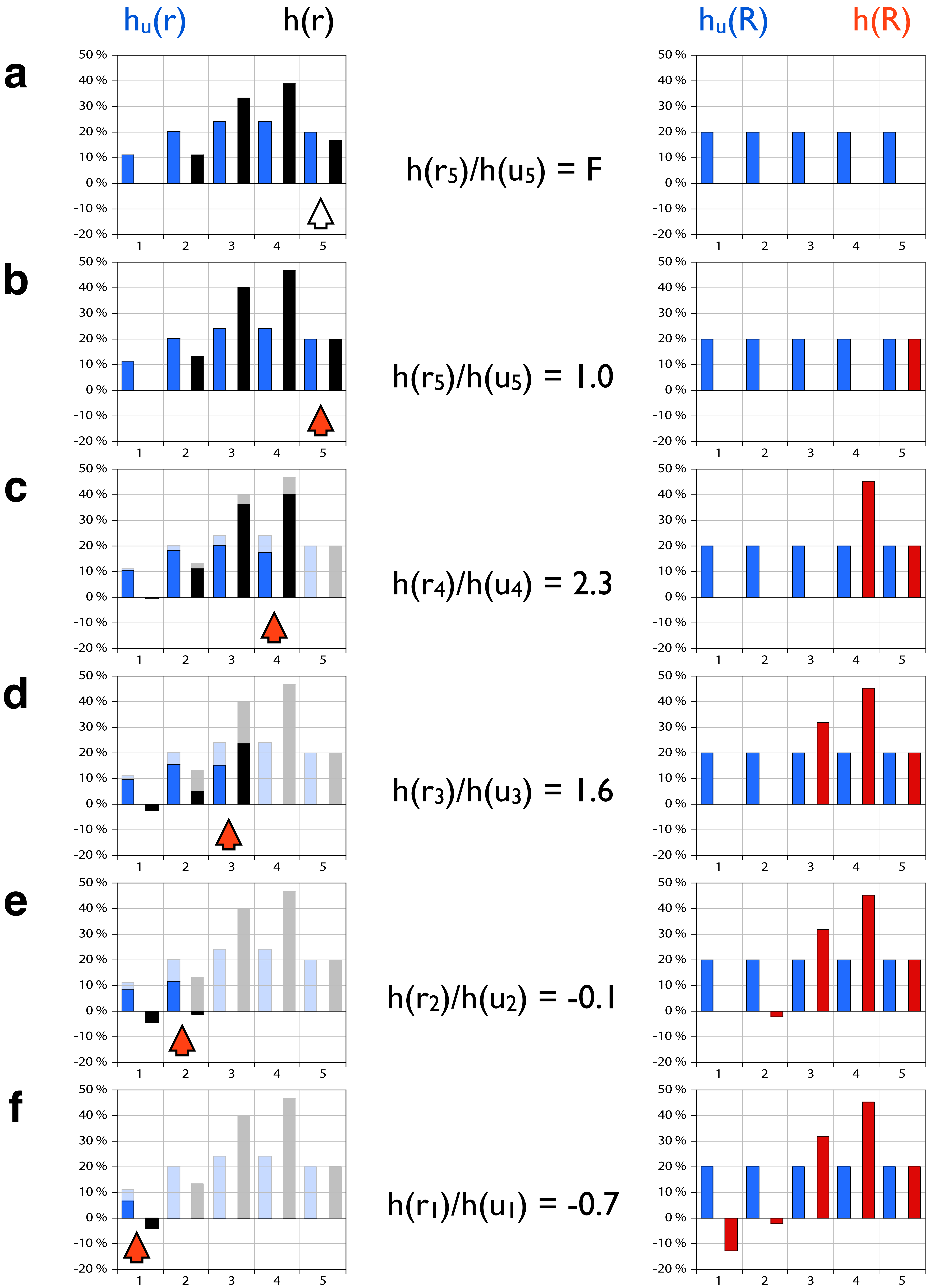

Figure 12.7
From 2-D to 3-D.
Procedure for conversion of distribution of circles, h(r), to parent distribution of spheres, h(R); example for measured h(r) with n = 5 bins: (a) before start, determine $f = h_u(r_5) / h(r_5)$, recalculate h(r) as $f \cdot h(r)$; (b) $F_1 = h(r_5)/h_u(r_5) = 1.00$, set $h(R_5)$ to $F_1 \cdot h_u(R_5)$, calculate $h_d(r)$ for size class $h(R_5)$ subtract from h(r), calculate $h_d(r)$ for size class $h_u(R_5)$, subtract from $h_u(r)$; (c) $F_2 = h(r_4)/h_u(r_4)$, set $h(R_4)$ to $F_2 \cdot h_u(R_4)$, calculate $h_d(r)$ for size class $h(R_4)$ subtract from h(r), calculate $h_d(r)$ for size class $h_u(R_4)$, subtract from $h_u(r)$; (d) $F_3 = h(r_3)/h_u(r_3)$, set $h(R_3)$ to $F_3 \cdot h_u(R_3)$, calculate $h_d(r)$ for size class $h(R_3)$ subtract from h(r), calculate $h_d(r)$ for size class $h_u(R_3)$, subtract from $h_u(r)$; (e) $F_4 = h(r_2)/h_u(r_2)$, set $h(R_2)$ to $F_4 \cdot h_u(R_2)$, calculate $h_d(r)$ for size class $h(R_2)$ subtract from h(r), calculate $h_d(r)$ for size cla ss $h_u(R_2)$, subtract from $h_u(r)$; (f) $F_5 = h(r_1)/h_u(r_1)$, set $h(R_1)$ to $F_5 \cdot h_u(R_1)$, calculate $h_d(r)$ for size class $h(R_1)$ subtract from h(r), calculate $h_d(r)$ for size class $h_u(R_1)$, subtract from $h_u(r)$; arrows point to bins from which ratios (center) are calculated.

```
 ***  stripstar  ***                          2012-aurora, rh
 -----------------------------------------------------------
 this program derives a possible distribution of spheres
 from measured distributions of sectional areas.
 it requires input in the form of binned data:
 histogram h(r): r = radius;  h = number frequency;
 --------------------------------------------------------
indicate if input is manual (0) or by file (1) >
[1] 0
indicate number of classes of histogram
h(r) (up to 20) >
[2] 5
indicate class width of h(r) (mm/inch/units/...) >
[3] 1
type 20 input frequencies (# or %, from smallest to largest)
bin no. 1:
[4] 0
bin no. 2:
[...] 20
bin no. 3:
60
...
etc.
```

```
indicate if input is manual (0) or by file (1>
[1] 1
file must contain list of h(r)
line 1: no. of bins (max. = 20), width of bin
line 2 ff.: h(r)

name of input file >
[2a] five.in
```

```
largest no-zero bin is h( 5)

 matrix r(i,j):
 i (row)    = size of section,
 j (column) = produced by size of sphere

             1       2       3       4       5
   1   0.20000 0.05359 0.03431 0.02540 0.02020
   2   0.00000 0.34641 0.11847 0.08178 0.06328
   3   0.00000 0.00000 0.44721 0.16367 0.11652
   4   0.00000 0.00000 0.00000 0.52915 0.20000
   5   0.00000 0.00000 0.00000 0.00000 0.60000

h(r) for uniform h(r): h(r)i =
(horizontal) sum (r)i,j=1,n
    1         0.33351
    2         0.60994
    3         0.72740
    4         0.72915
    5         0.60000

name of output file
[5] five.out
```

Software Box 12.1
Dialog with program STRIPSTAR; answers are numbered and highlighted, see text for explanation.

```
r=radius of sections, R=radius of spheres,
h=frequency, v=volume fraction


           calc. distributions:        comparison:
           spheres    sph.& antisph.   rel.input r      recalc.r
  class    h(R)       h*(R)            (h( 5) = 1.00):  from h(R):


  1         0.00000     -0.63339           0.00000         0.22078
  2         0.00000     -0.10806           0.66667         0.72905
  3         1.59279      1.59279           2.00000         2.00000
  4         2.26779      2.26779           2.33333         2.33333
  5         1.00000      1.00000           1.00000         1.00000


                      spheres only            spheres & antispheres
      r   h(r)(%)     h(R)(%)     v(R)(%)     h*(R)(%)        v*(R)(%)

  1.000     0.00        0.00        0.00       -11.31          -0.20
  2.000    11.11        0.00        0.00        -1.93          -0.27
  3.000    33.33       32.77       13.73        28.43          13.67
  4.000    38.89       46.66       46.35        40.48          46.13
  5.000    16.67       20.57       39.92        17.85          39.73
```

Software Box 12.1
(rigt side)

a

```
5, 1
0
20
60
70
30
```

b

r=radius of sections, R=radius of spheres, h=frequency, v=volume fraction

		spheres only		spheres & antispheres	
r	h(r)(%)	h(R)(%)	v(R)(%)	h*(R)(%)	v*(R)(%)
1.000	0.00	0.00	0.00	-11.31	-0.20
2.000	11.11	0.00	0.00	-1.93	-0.27
3.000	33.33	32.77	13.73	28.43	13.67
4.000	38.89	46.66	46.35	40.48	46.13
5.000	16.67	20.57	39.92	17.85	39.73

Software Box 12.2

STRIPSTAR input and output:

(a) Input file: first line, first entry = number of data points; first line, second entry = interval of radius; following lines = entries for h(r);

(b) result file:

h(R), v(R) = number weighted and volume weighted histogram of calculated radii of equivalent spheres, using positive values only;

h*(r), h*(R) = same as h(R) and v(R), including negative values ('antispheres').

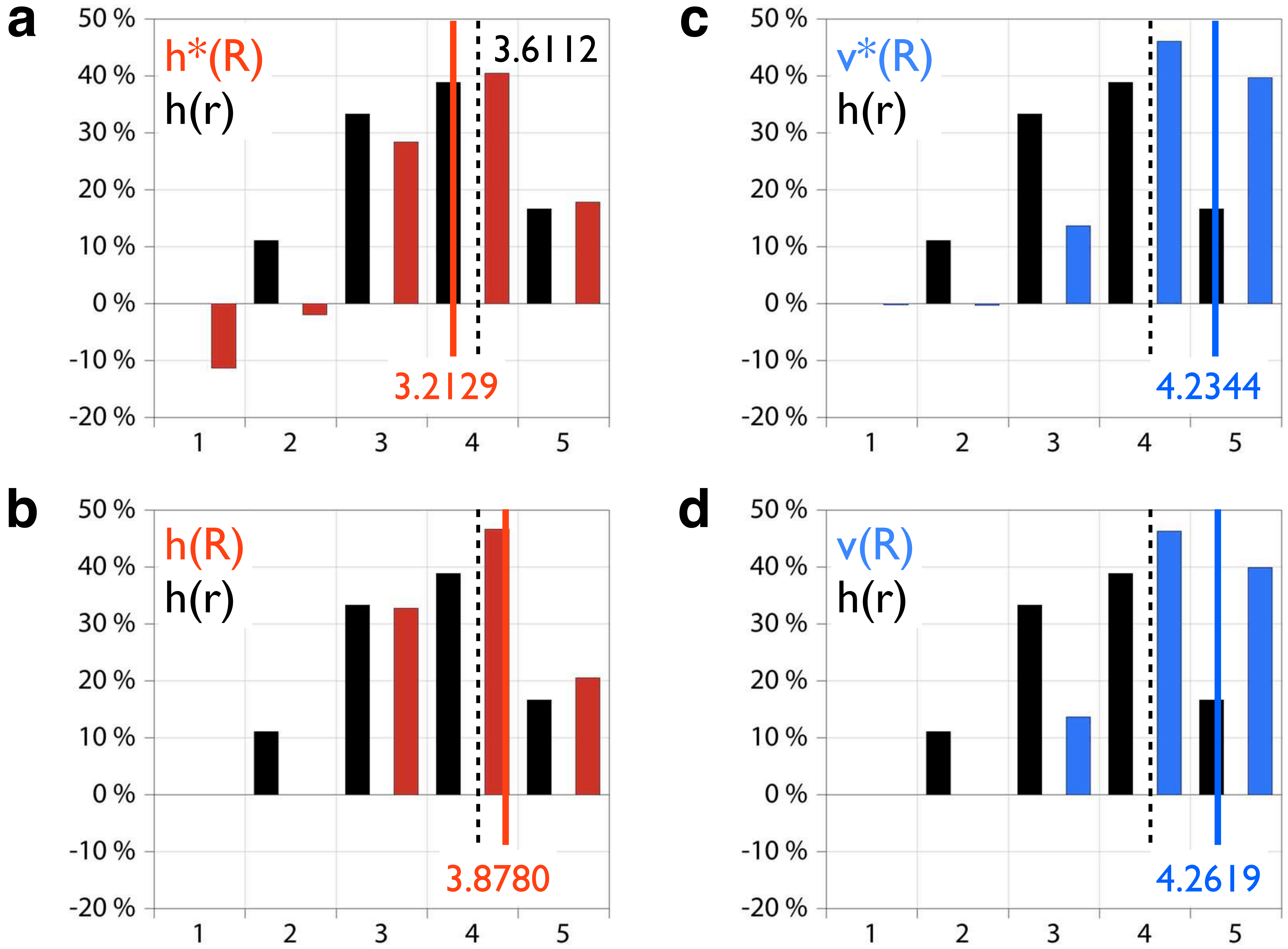

Figure 12.8
STRIPSTAR results.
From the measured distribution of sectional circles, h(r) (in black), a number of results are derived:
(a) numerical density histogram of h*(R) of spheres including 'antispheres';
(b) h(R) = same as (a), using only positive frequencies;
(c) volumetric density histogram v*(R) of spheres including 'antispheres';
(d) v(R) = same as (c), using only positive frequencies;
stippled lines: mean value of h(r); solid lines: mean values of h*(R), h(R), v*(R) and v(R).

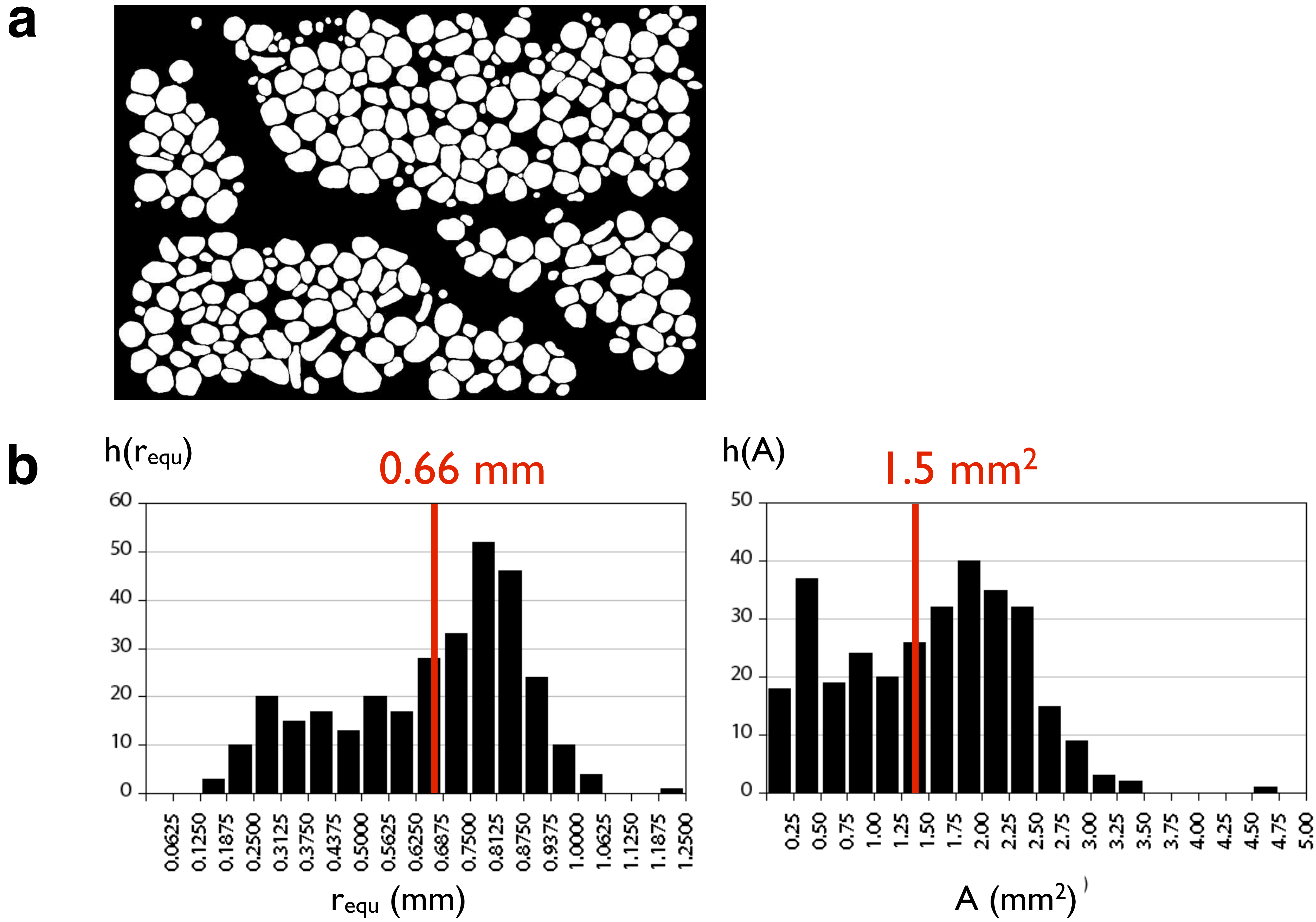

Figure 12.9
2-D input data for grain size analysis.
(a) Bitmap of oolithic limestone, total number of ooides = 313;
(b) h(r_{equ}) of equivalent radii, r_{equ}, mean indicated in red;
(c) h(A) of measured cross sectional areas, A, mean indicated in red.
Results are shown in Fig.12.10.

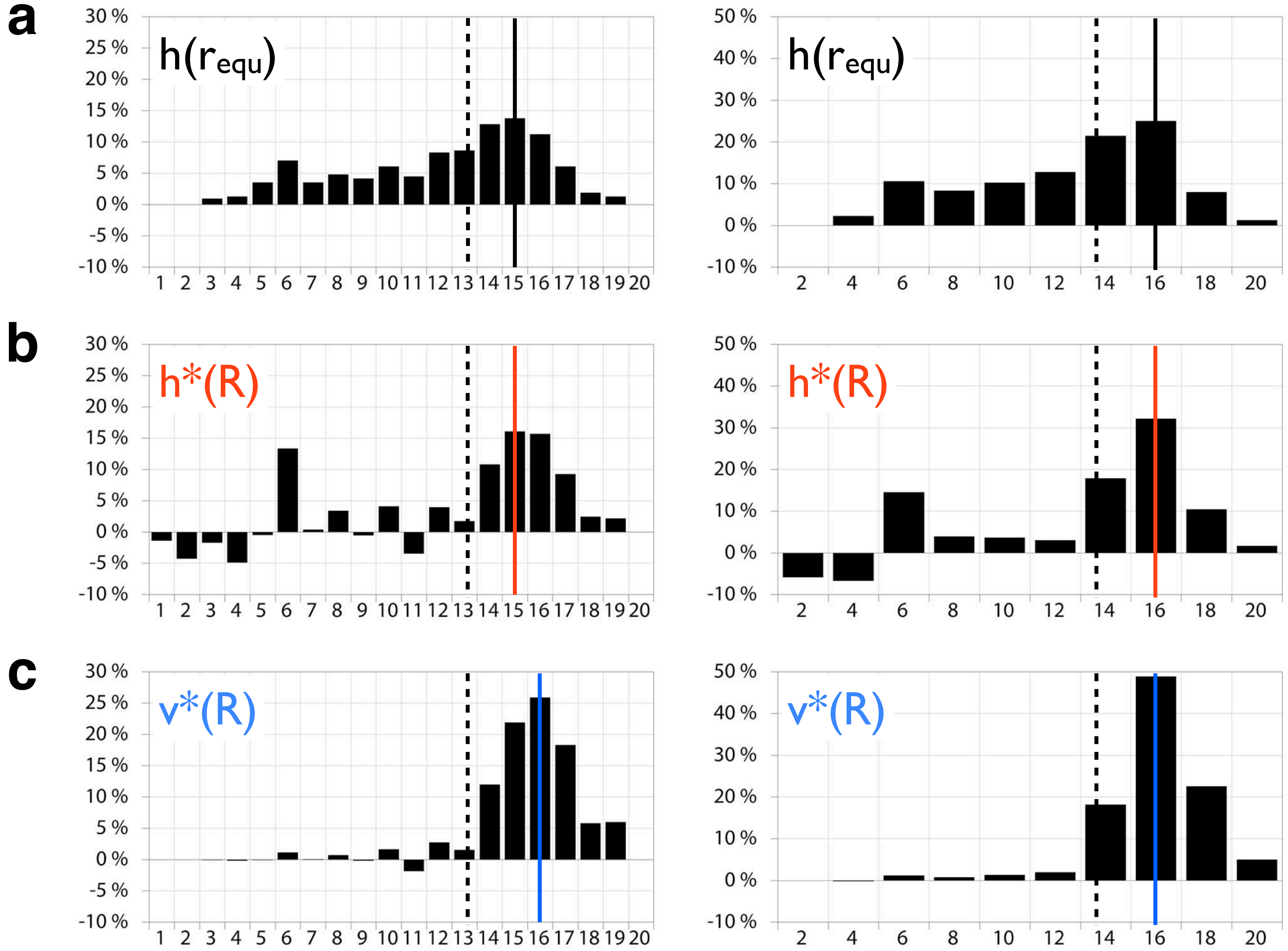

Figure 12.10
3-D grain size analysis of oolithic limestone.
Input data is histogram of 2-D radii shown in Figure 12.9.b.
(a) Measured size distribution of equivalent radii, $h(r_{equ})$, size is in pixels;
(b) derived distribution, $h^*(R)$, numerical densities;
(c) derived distribution, $v^*(R)$, volumetric densities;
left: 10 bins, right: 20 bins.

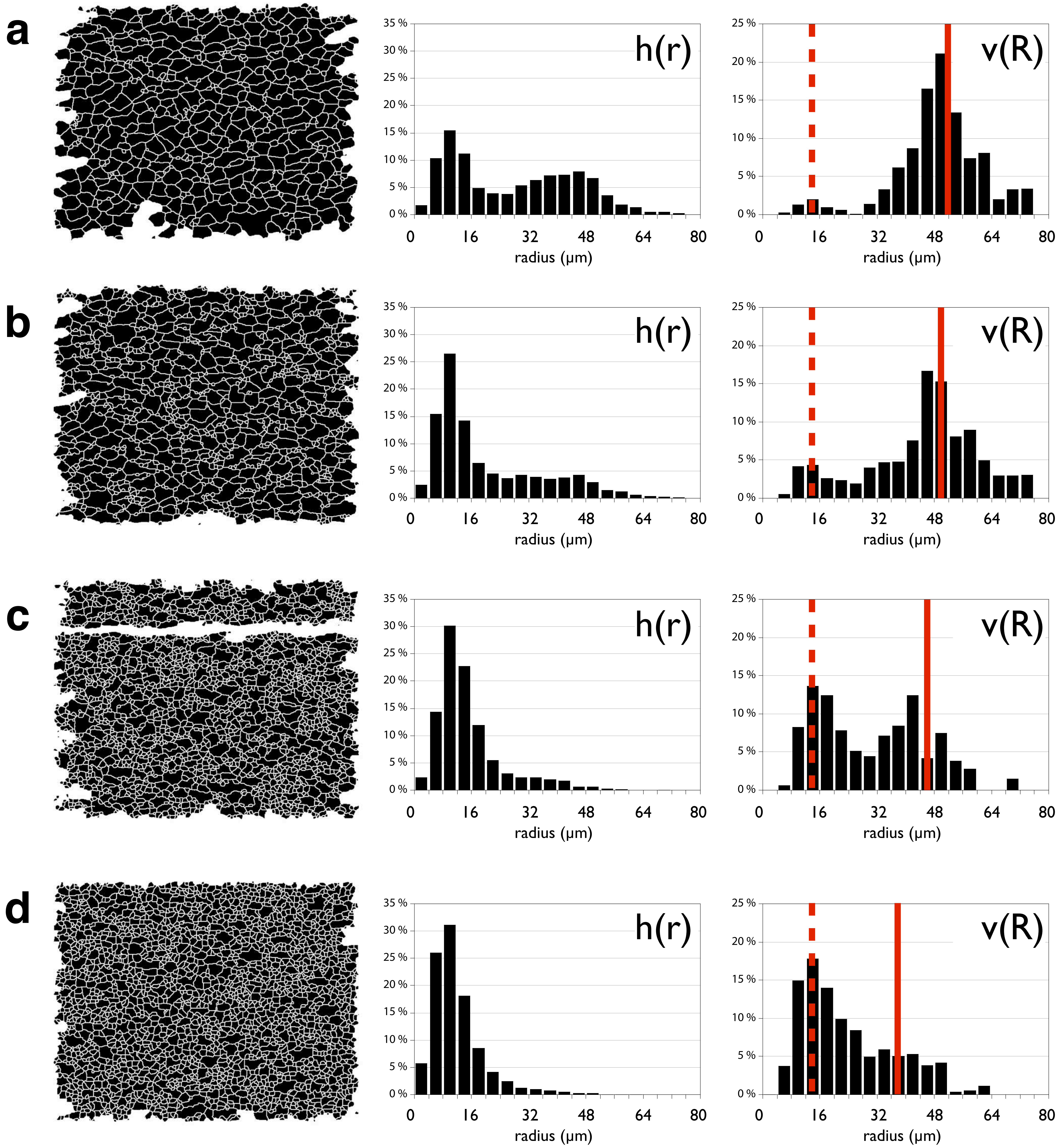

Figure 12.11
Grain size analysis of dynamically recrystallized quartzite.
Grain maps of vertically compressed sample (left), histogram, h(r), of radii of area equivalent circles (center), and histogram, v(R), of radii of volume equivalent spheres (right), for increasing recrystallization (compare Figure 11.9):
(a) sample site A: ~10 %;
(b) sample site B: ~25 %;
(c) sample site C: ~50 %;
(d) sample site D: ~75 %.

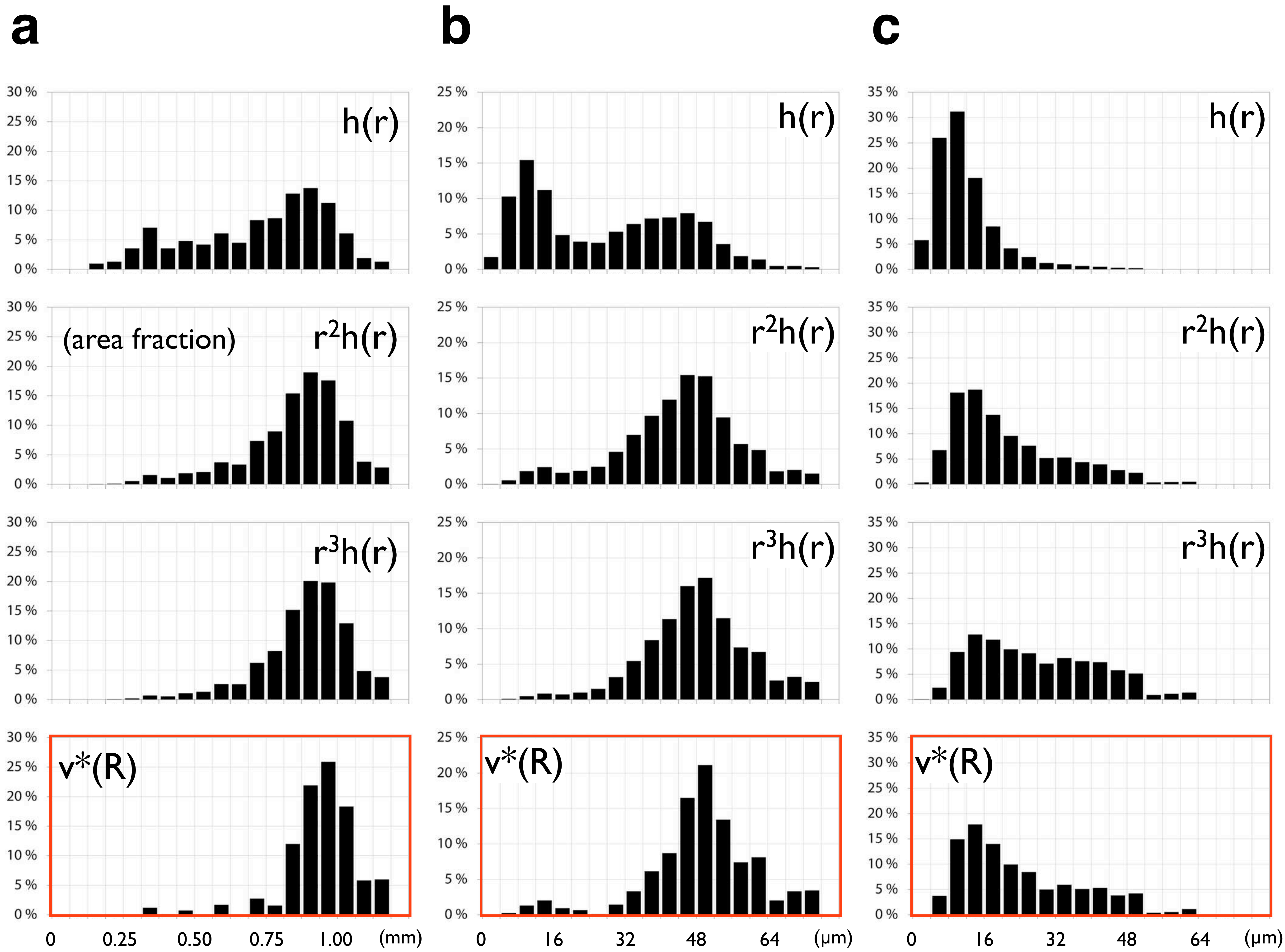

Figure 12.12
Shortcuts.
Approximations to a 3-D grain size determination are calculated for:
(a) oolithic limestone (Figure 12.10);
(b) experimentally deformed quartzite, 10% recrystallized (see Figure 12.11.a);
(c) same as (b), 75% recrystallized (see Figure 12.11.d);
from top top bottom:
h(r) = measured distribution of area equivalent circles;
$r^2 \cdot h(r)$ = distribution of areas calculated from h(r);
$r^3 \cdot h(r)$ = distribution of volumes calculated from h(r);
v*(R) = distribution of volumes calculated by STRIPSTAR.

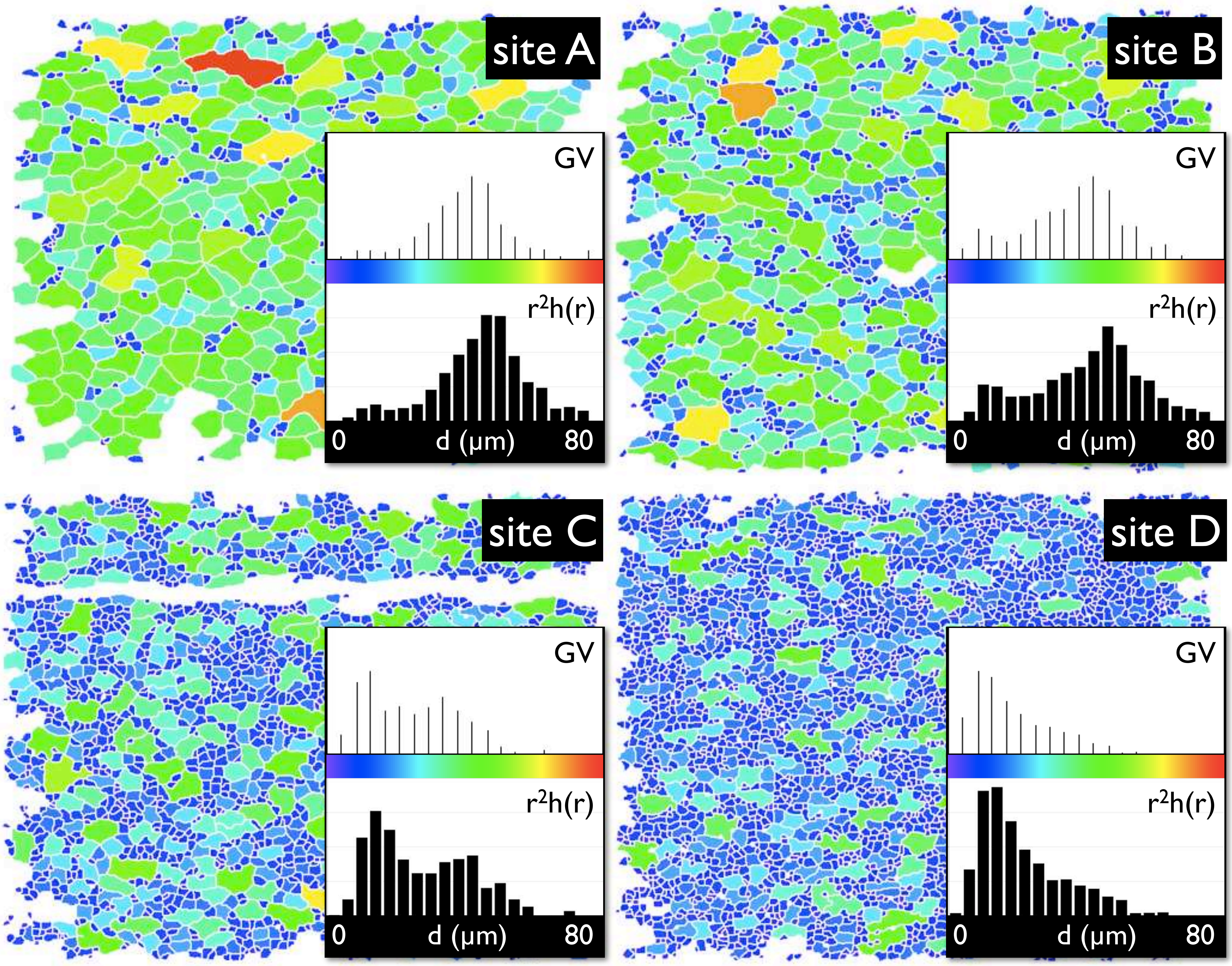

Figure 12.13
Histograms from grain size maps.
Grain size maps for the samples site A to D (Figure 12.11); color coding by the 'Rainbow' LUT (blue = 0, red = 255); number of distinct gray levels set to 20.
GV = histogram of gray value = histogram of area fraction of size class;
$r^2 \cdot h(r)$ = distribution of areas calculated from h(r); compare to results in second row of Figure 12.12.

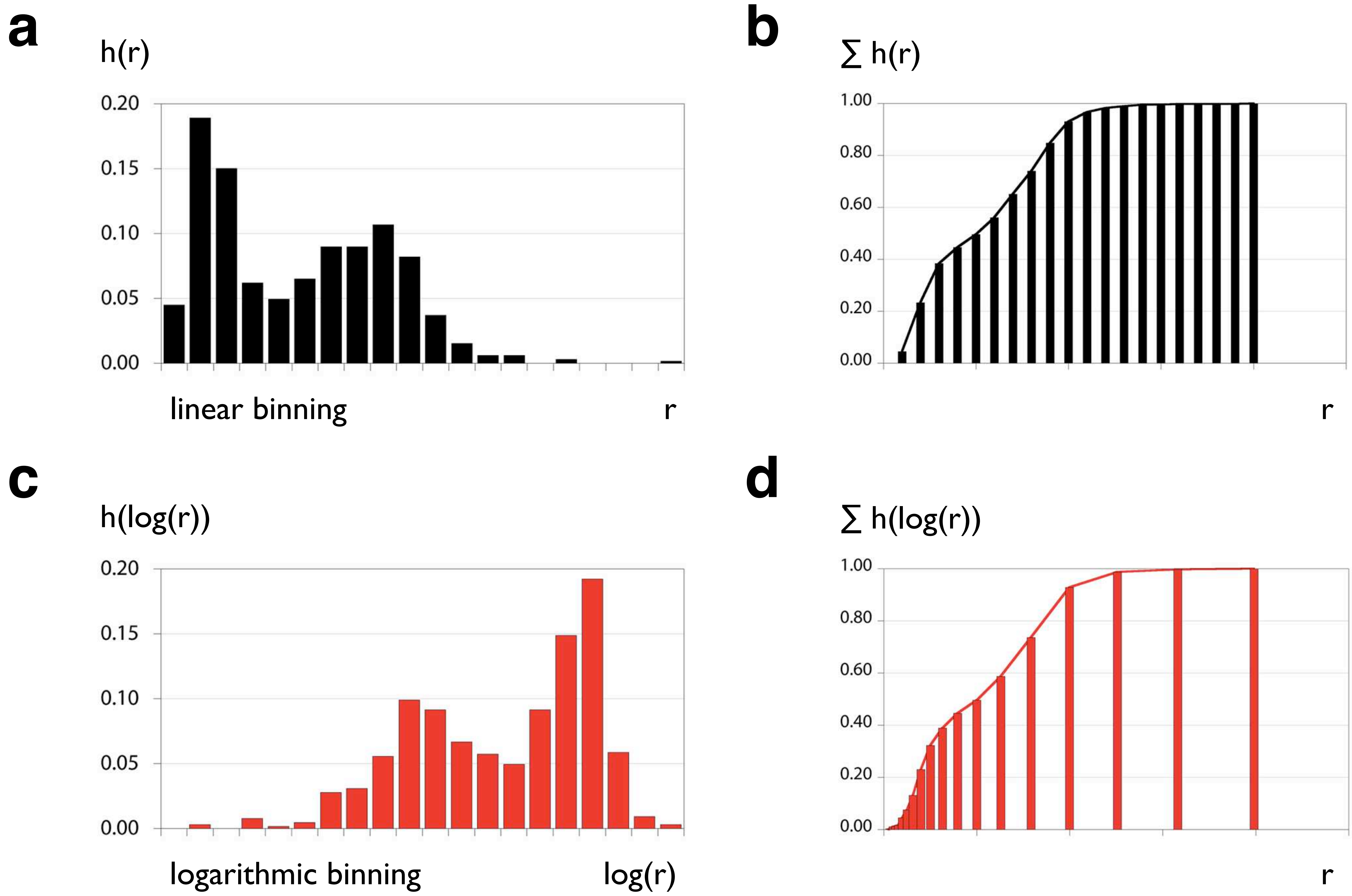

Figure 12.14
Linear versus logarithmic plots.
Histogram h(r) is from Figure 12.11.a.
(a) Linear binning, Δr = constant ⇒ histogram = h(r);
(b) cumulative histogram, $\sum h(r)$, for (a);
(c) logarithmic binning: $\Delta \log(r)$ = constant ⇒ histogram = h(log(r));
(d) cumulative histogram, $\sum h(\log(r))$, for (c).

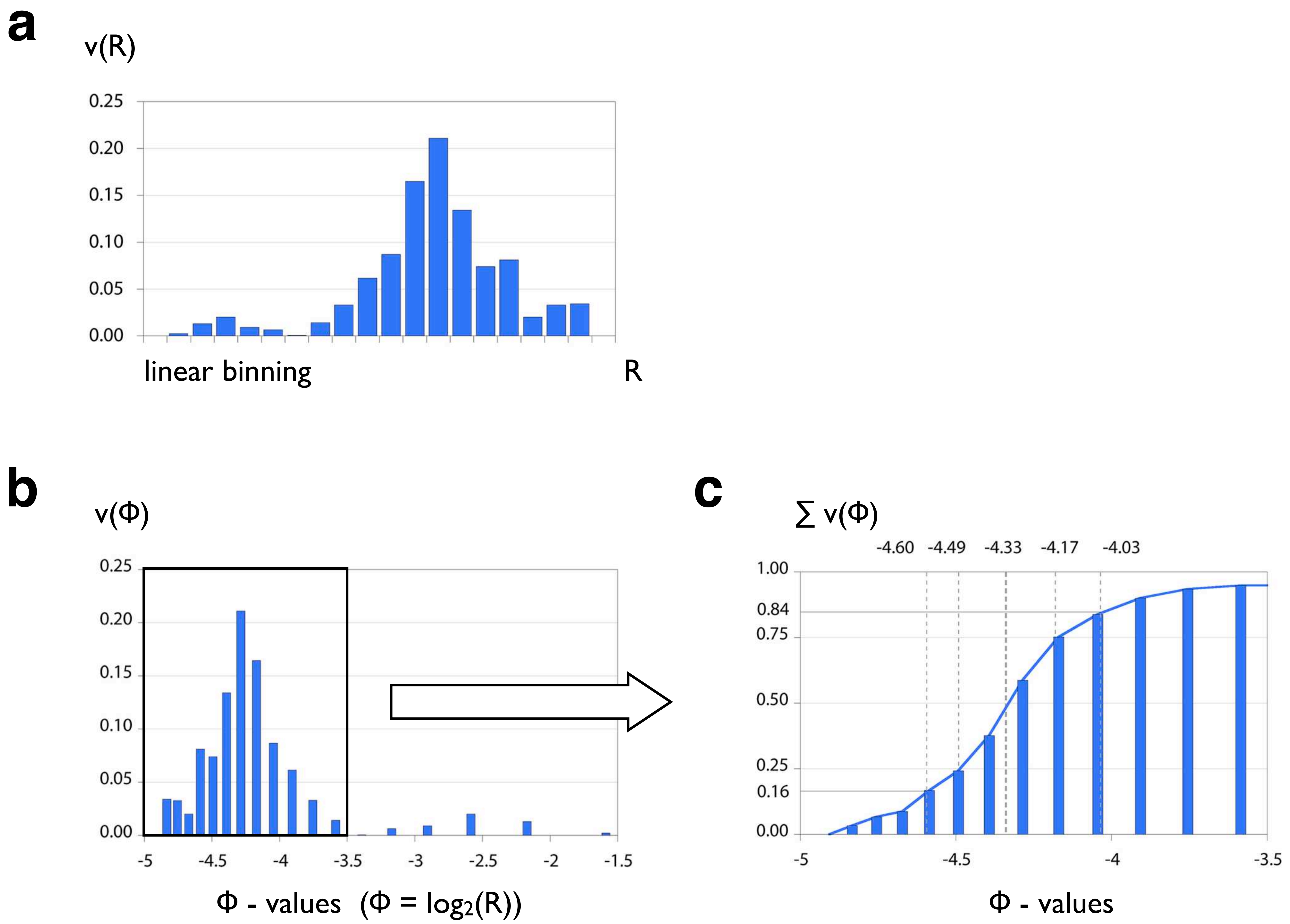

Figure 12.15

Deriving plots of Phi-values for sieved grain sizes.

(a) Volumetric histogram, v(R), obtained by STRIPSTAR; result v(R) is from Figure 12.11.a;

(b) v(R) plotted as v(Φ) for $\Phi = -\log_2(R)$, where R is in mm;

(c) cumulative histogram $\sum v(\Phi)$ for (b), central section; quartile values, Φ_{25}, Φ_{50} (=median) and Φ_{75}, and16th and 84th percentile, Φ_{16} and Φ_{84}, are indicated on top.