

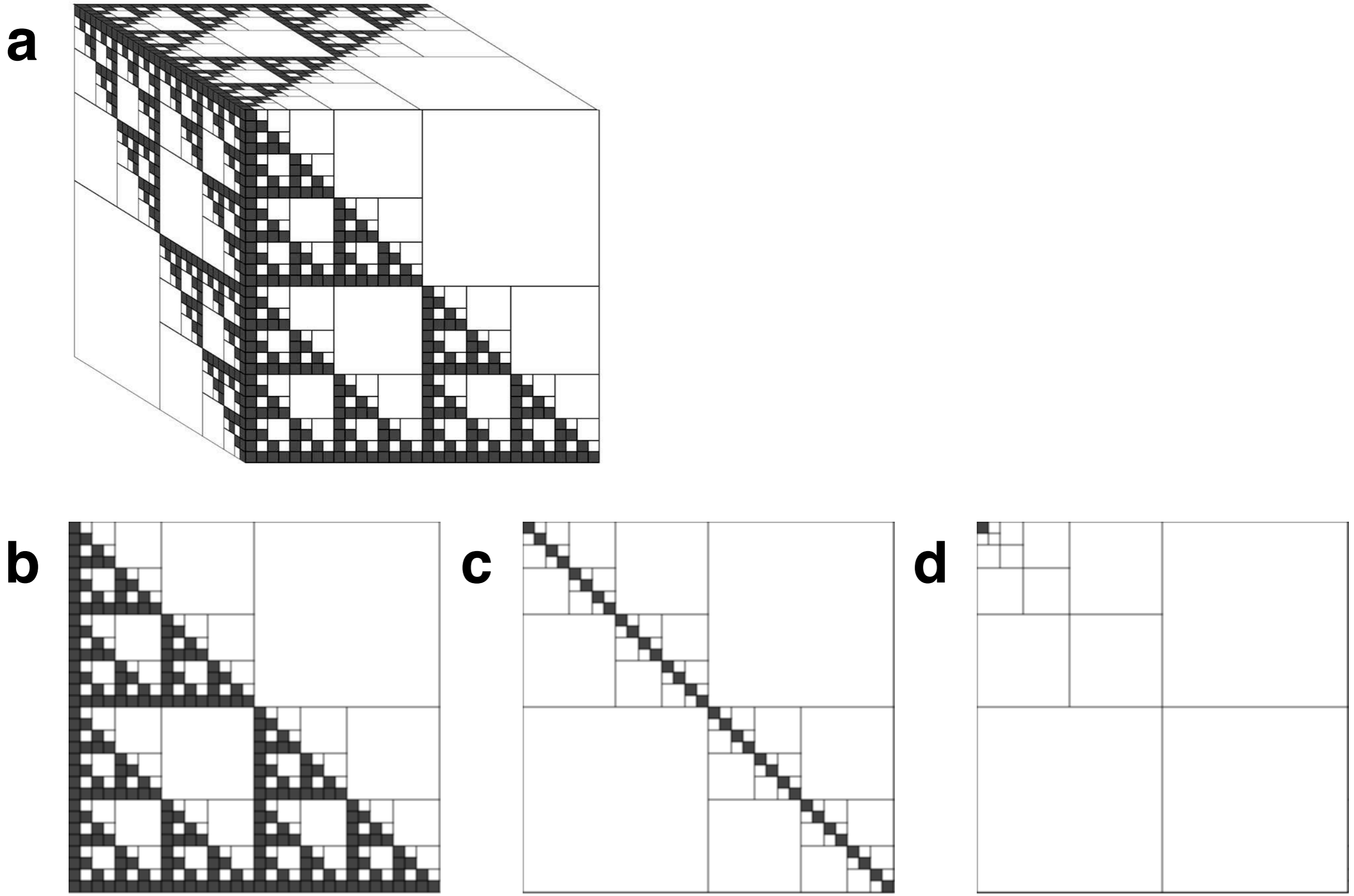
**Figure 13.1**

Self-similarity at various scales

Various types of fault rock, shown at increasing magnifications:

(a) shown in outcrop; (b) same as (a) in thin section; (c) thin section with angular (in-situ) fragments; (d) same as (c) with angular (in-situ) and rounded (dislocated) fragments; (e) SEM micrograph with (dislocated) angular fragments.





**Figure 13.2**

Fragmentation of a cube.

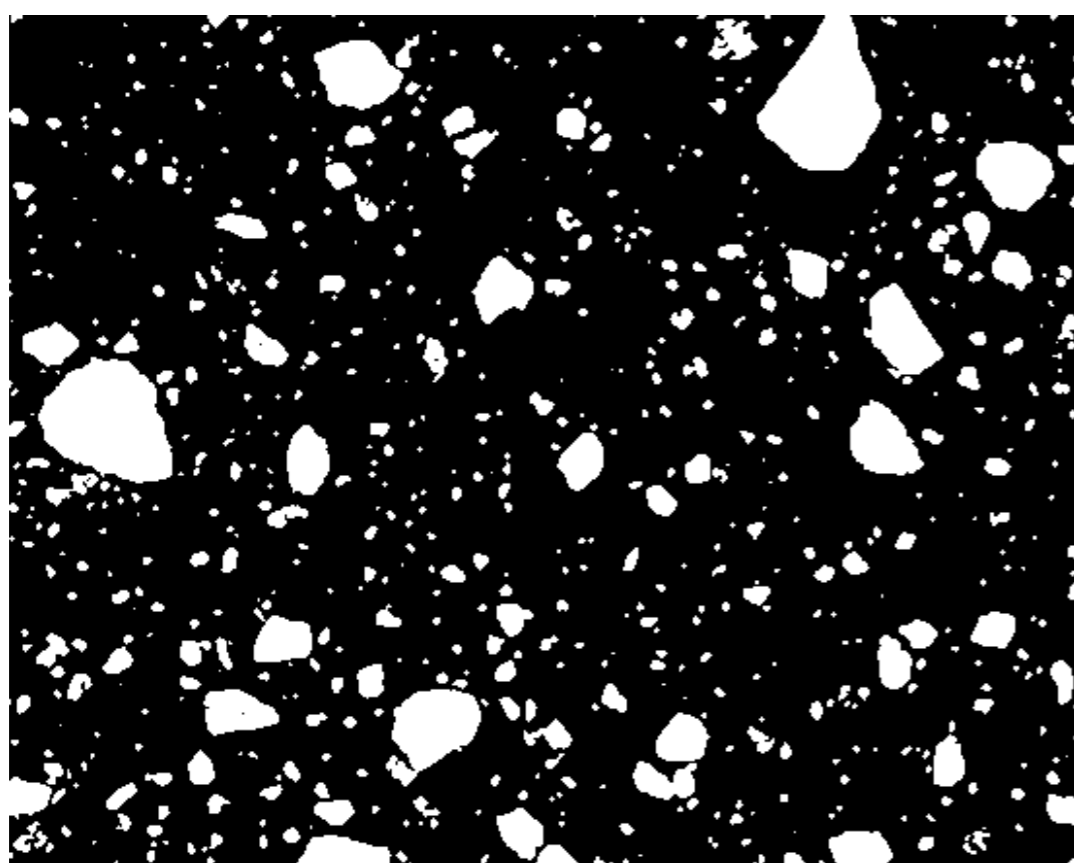
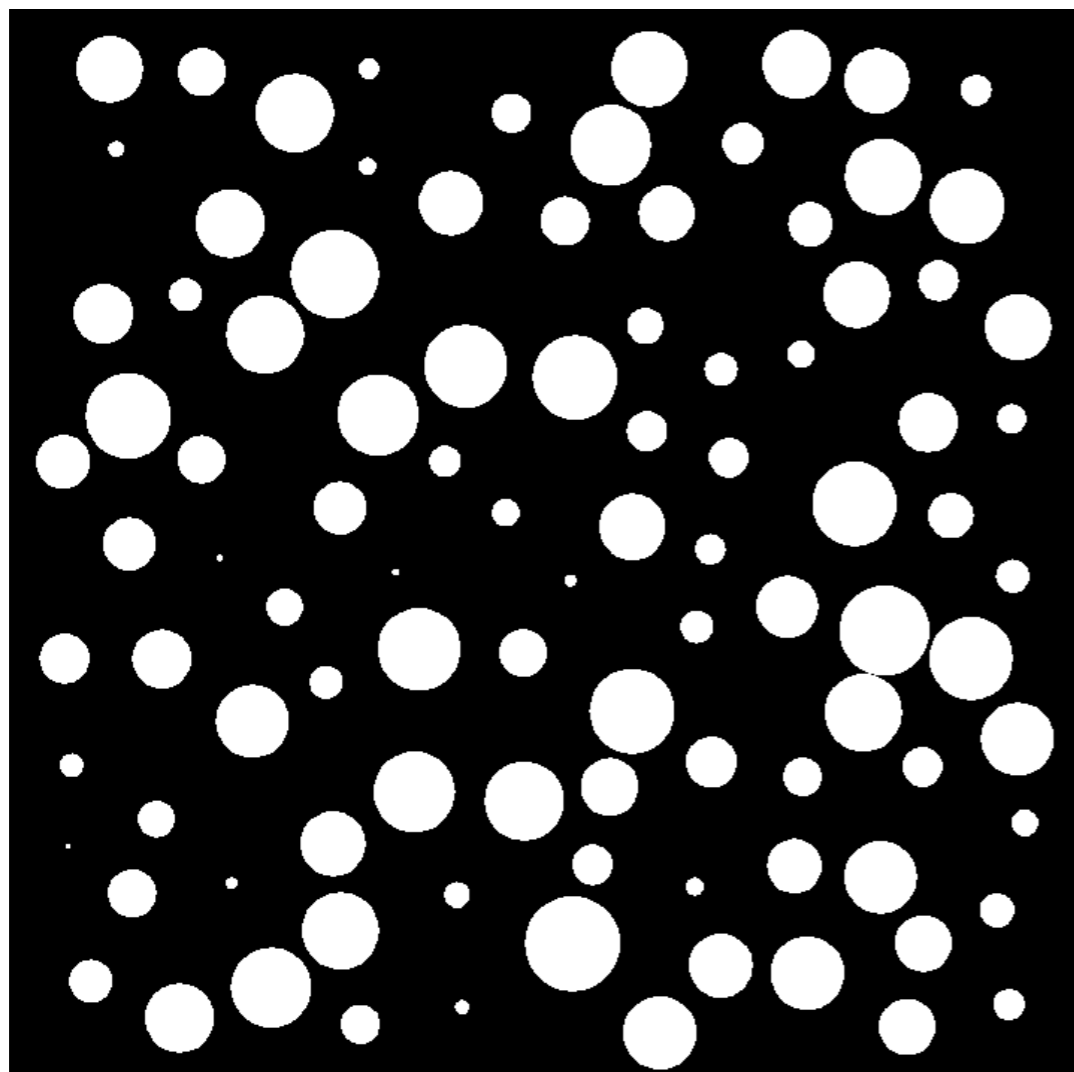
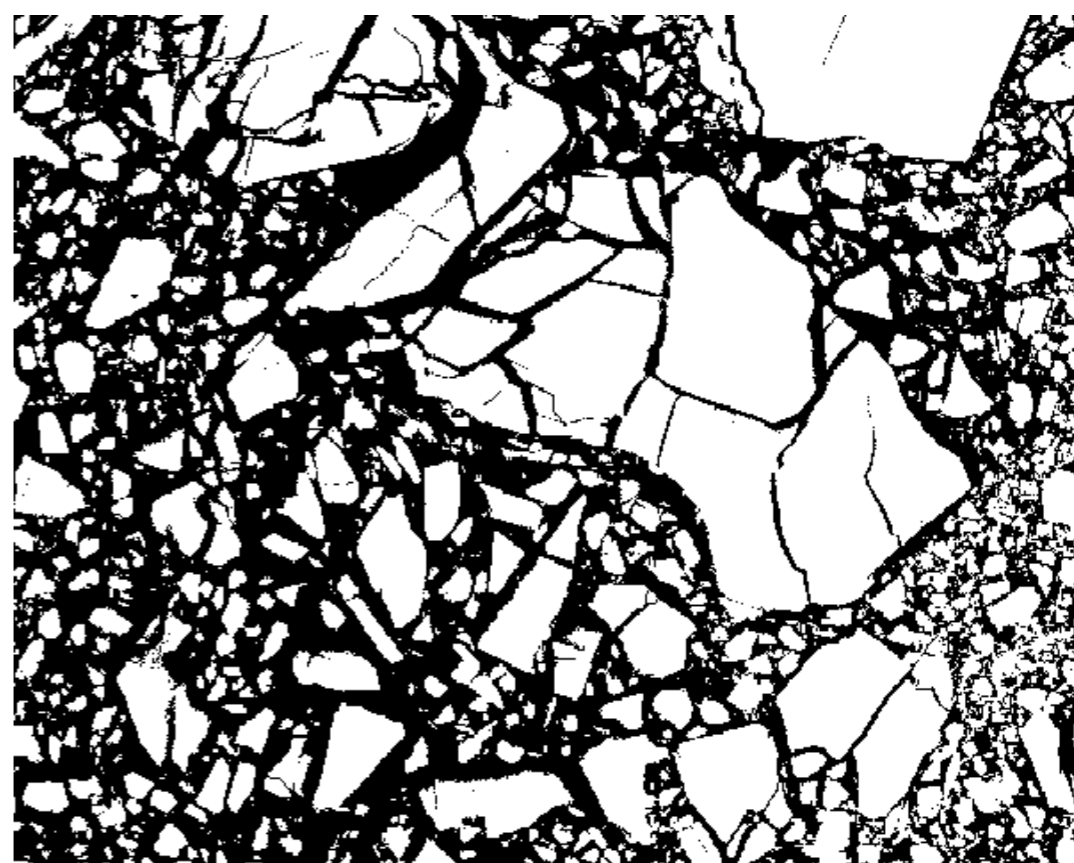
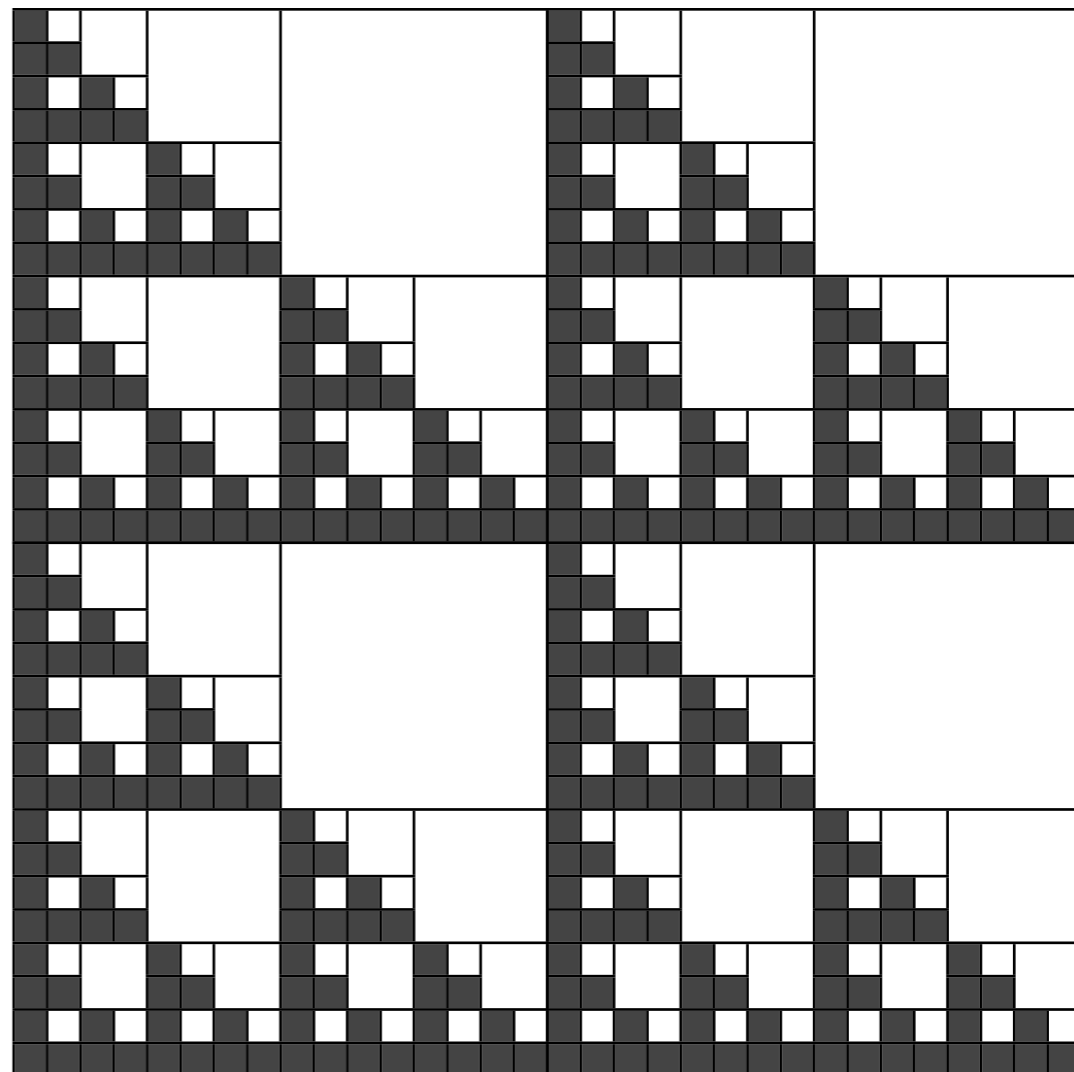
Fragmentation of the cube produces 8 cubes ( $F = 8$ ) of  $1/2$  the width of the original.

(a) Example of a cube where, at each scale, 2 out of 8 cubes remain unfragmented ( $N = 2$ ), while 6 are fragmented further;

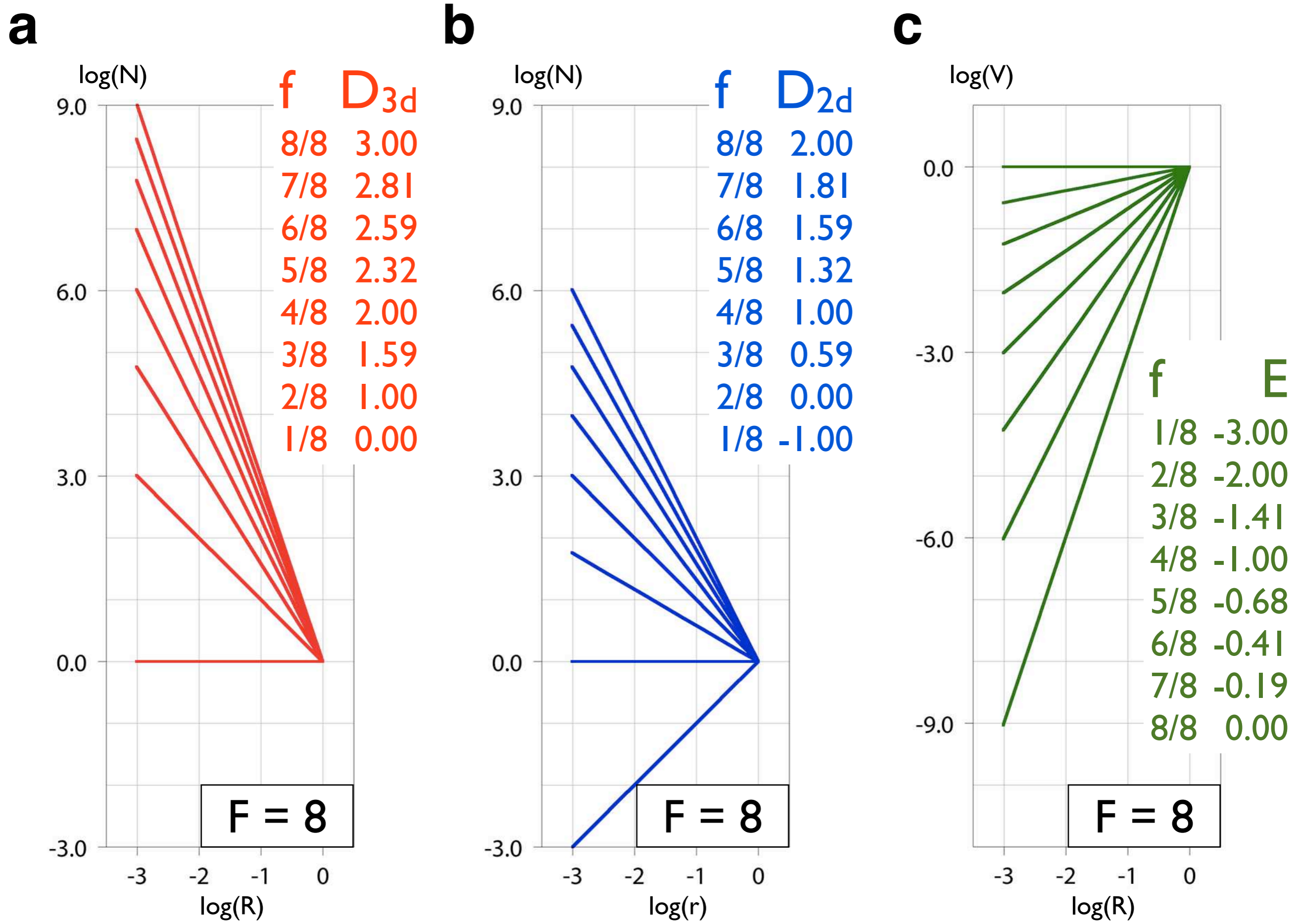
(b) front view of cube (a): the fragmentation fraction,  $f = (F - N) / F = 6 / 8$ ;

(c) front view of a cube with  $f = 4 / 8$ ;

(d) front view of a cube with  $f = 1 / 8$ .

**a****b****Figure 13.3**

Two conceptual models for grain size determination.  
(a) Stereological model: particles are diluted in matrix;  
(b) fractal model: fragments are densely packed.



**Figure I3.4**

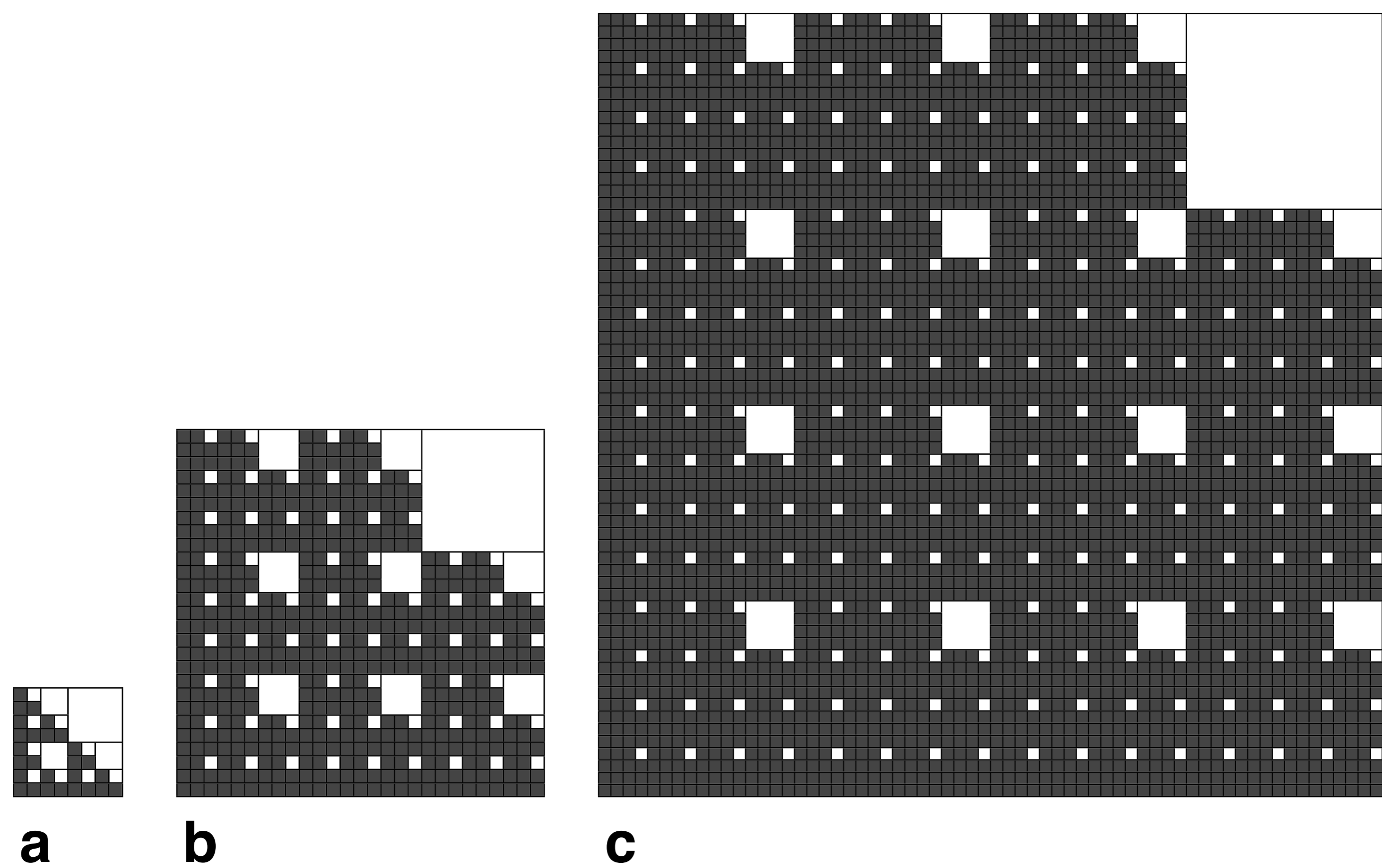
Fractal dimension of grain size distribution created by fragmenting a cube.

Fragmentation number,  $F$ , is 8 (as in Figure I3.2), fragmentation fractions,  $f$ , range from 1/8 to 7/8.

(a)  $\log(N)$ - $\log(r)$  plot, where  $N$  = number and  $R$  = size of fragments, the slope,  $D_{3d}$ , is the fractal dimension of the size distribution of 3-D grains;

(b)  $\log(N)$ - $\log(r)$  plot where  $N$  = number and  $r$  = size of cross sections of fragments, the slope,  $D_{2d}$ , is the fractal dimension of the size distribution of 2-D grains;

(c)  $\log(V)$ - $\log(r)$  plot where  $V$  = volume and  $R$  = size of fragments,  $E = D_{3d} - 3$ .



**Figure 13.5**

Higher fragmentation numbers.

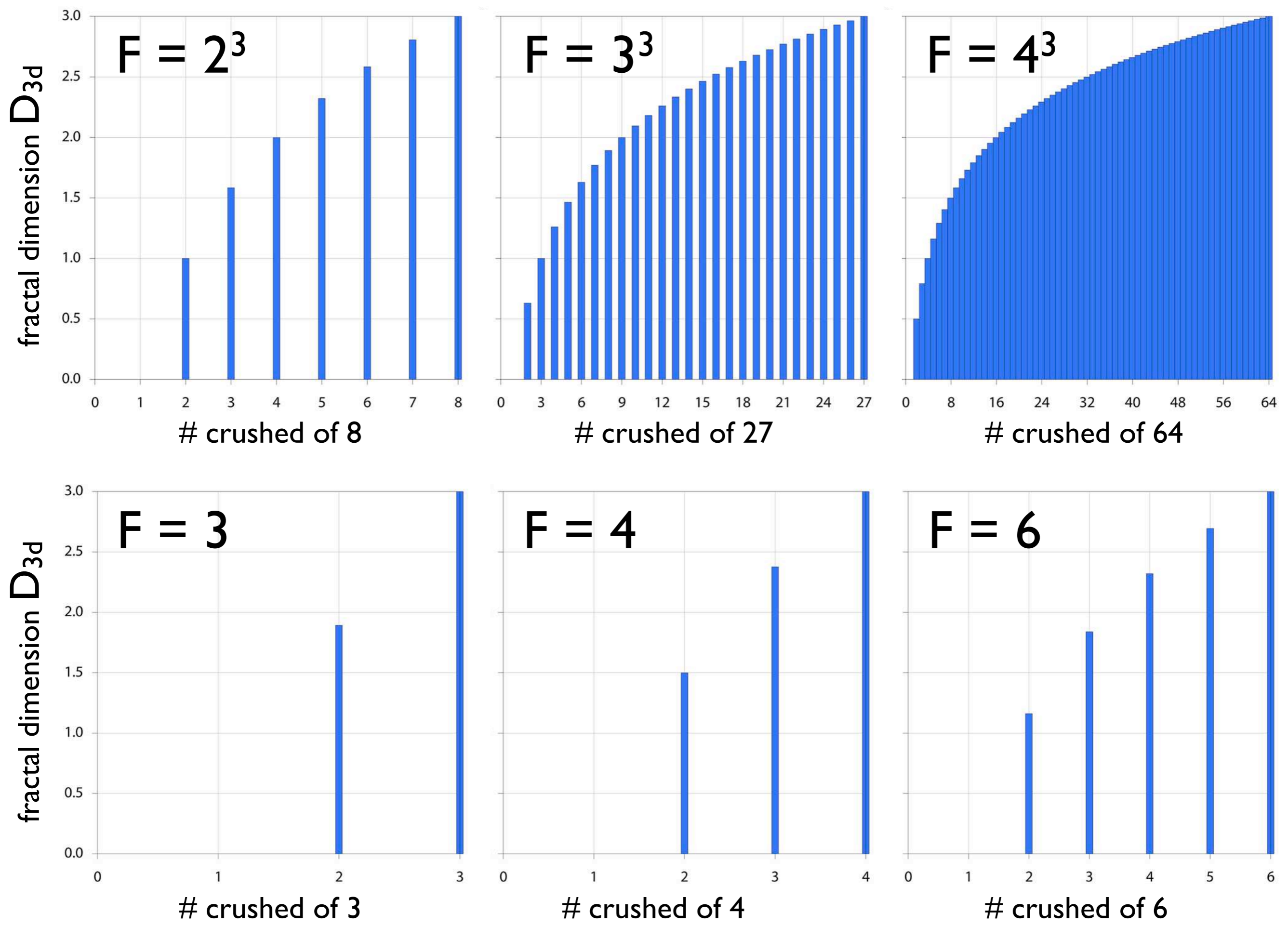
Front views of cubes with different fragmentation numbers,  $F$ , and different fragmentation fractions,  $f$ .

The cube is fragmented into:

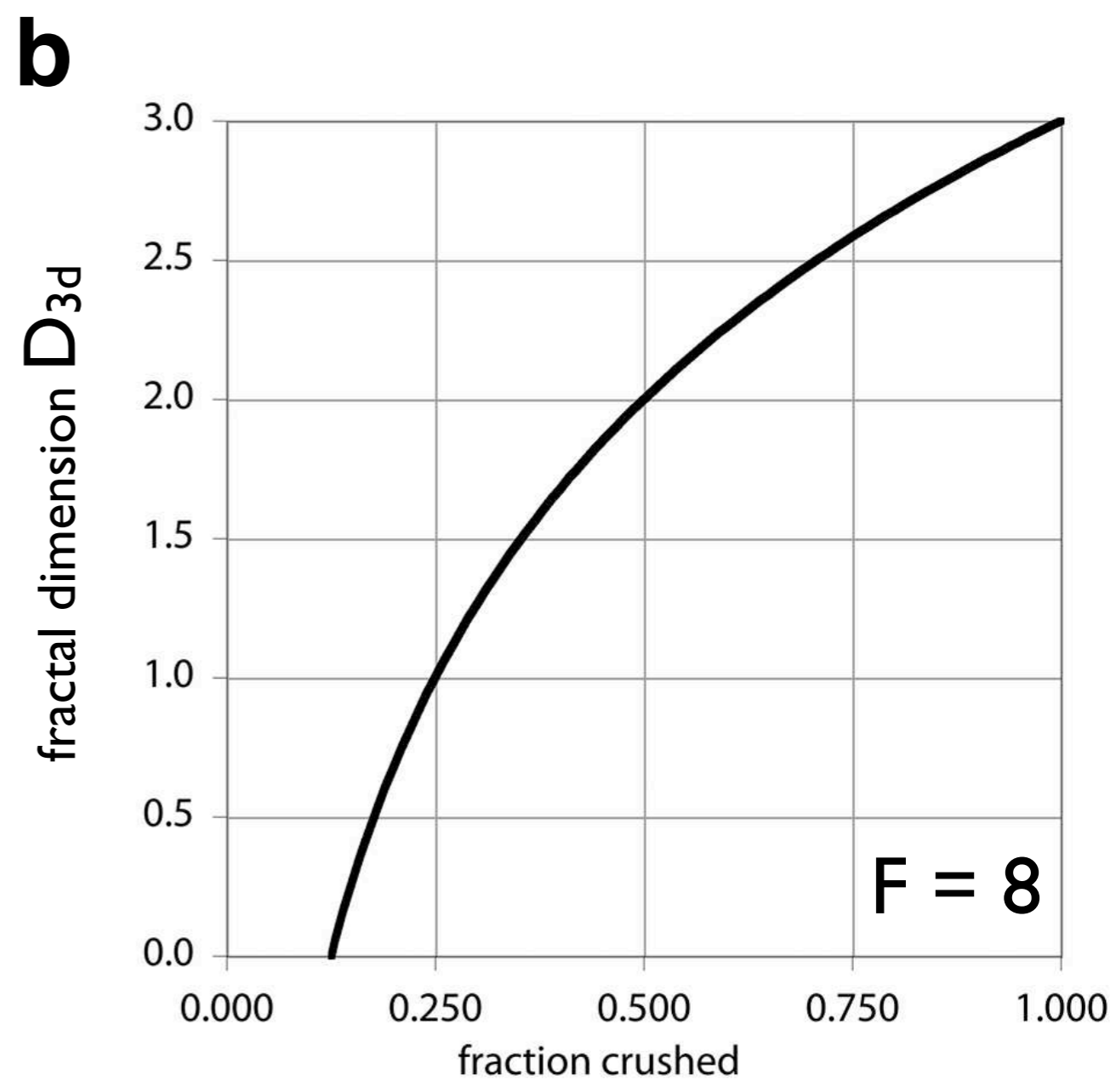
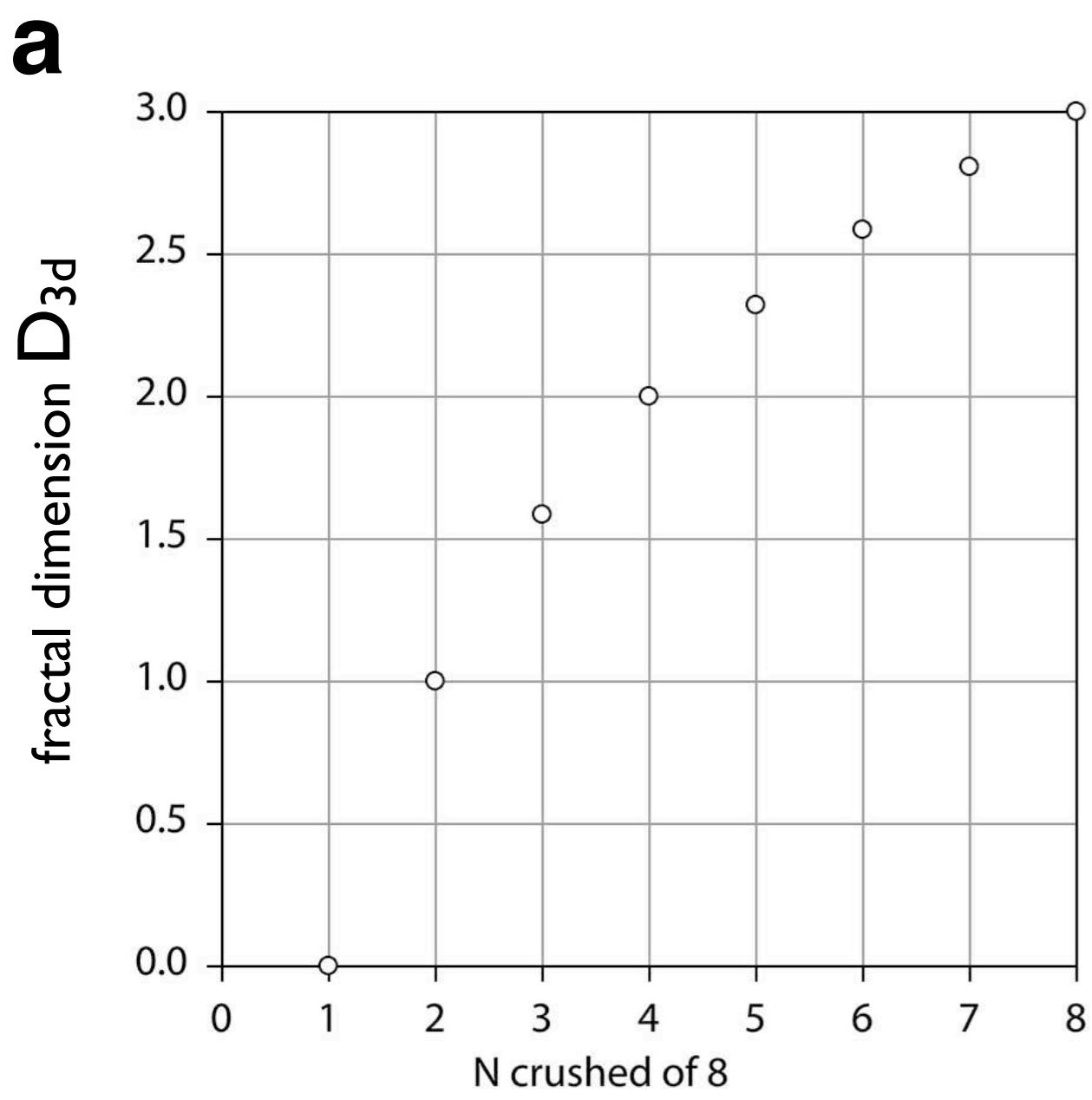
(a)  $F = 2 \cdot 2 \cdot 2 = 8$ ,  $N = 2$ ,  $f = 6 / 8$ ;

(b)  $F = 3 \cdot 3 \cdot 3 = 27$ ,  $N = 3$ ,  $f = 24 / 27$ ;

(c)  $F = 4 \cdot 4 \cdot 4 = 64$ ,  $N = 4$ ,  $f = 60 / 64$ .

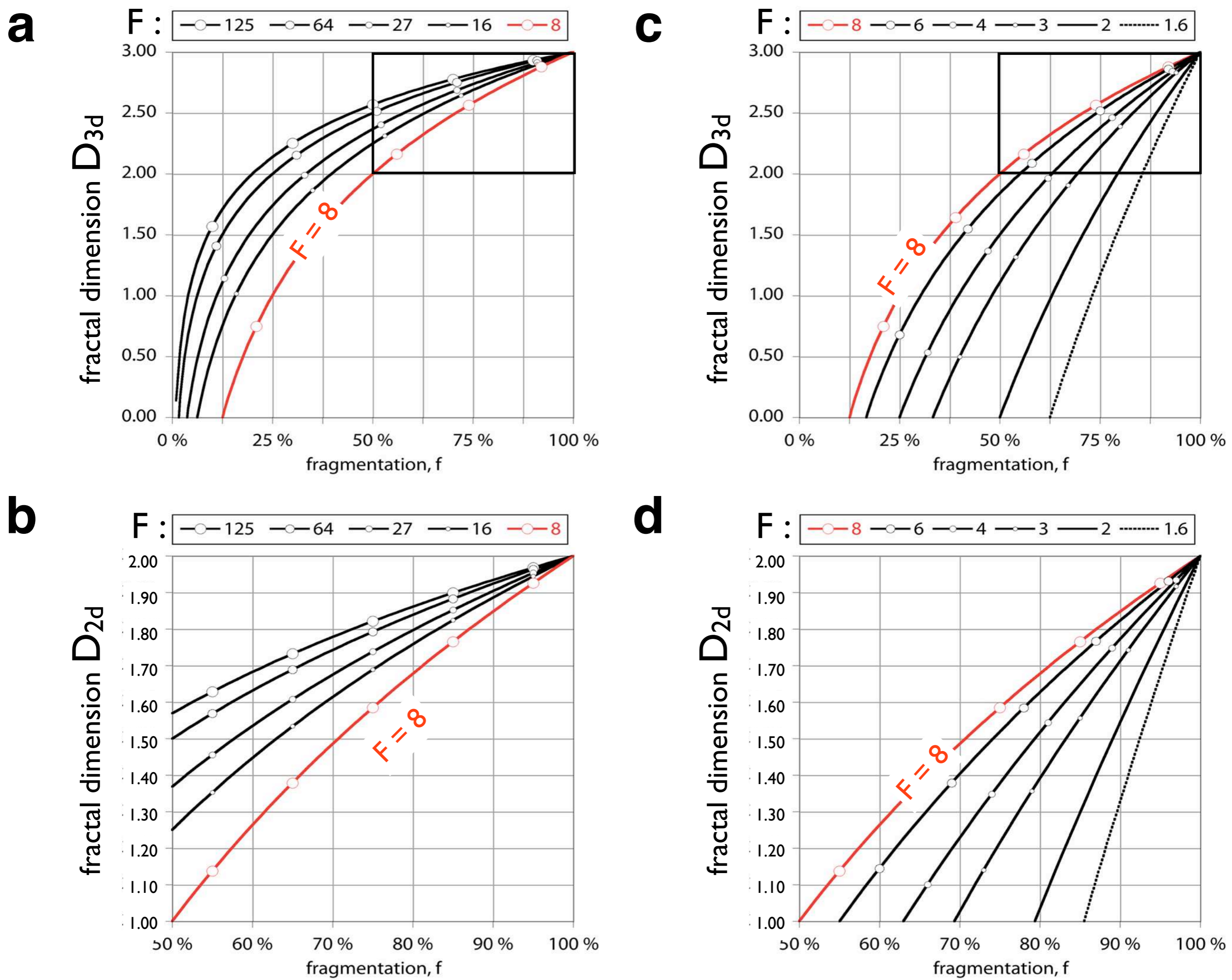


**Figure 13.6**  
 Fractal dimensions for different fragmentations.  
 Plots of fractal dimension,  $D_{3d}$ , versus number of crushed fragments, for different fragmentation numbers,  $F$ .



**Figure 13.7**  
 Fractal dimension for continuous fragmentation fraction,  $f$ .  
 Fragmentation number,  $F = 8$ .  
 (a) Discrete fragmentation fraction,  $f = (F - N_i) / F$  for  $i = 1$  to  $8$ ;  
 (b) same plot as (a) for continuously defined  $f$ :  $(0.00 \leq f \leq 1.00)$ .





**Figure 13.8**

Fractal dimension for continuously defined fragmentation numbers and fragmentation fractions.

(a)  $D_{3d}$  for fragmentation numbers  $F \geq 8$ ;

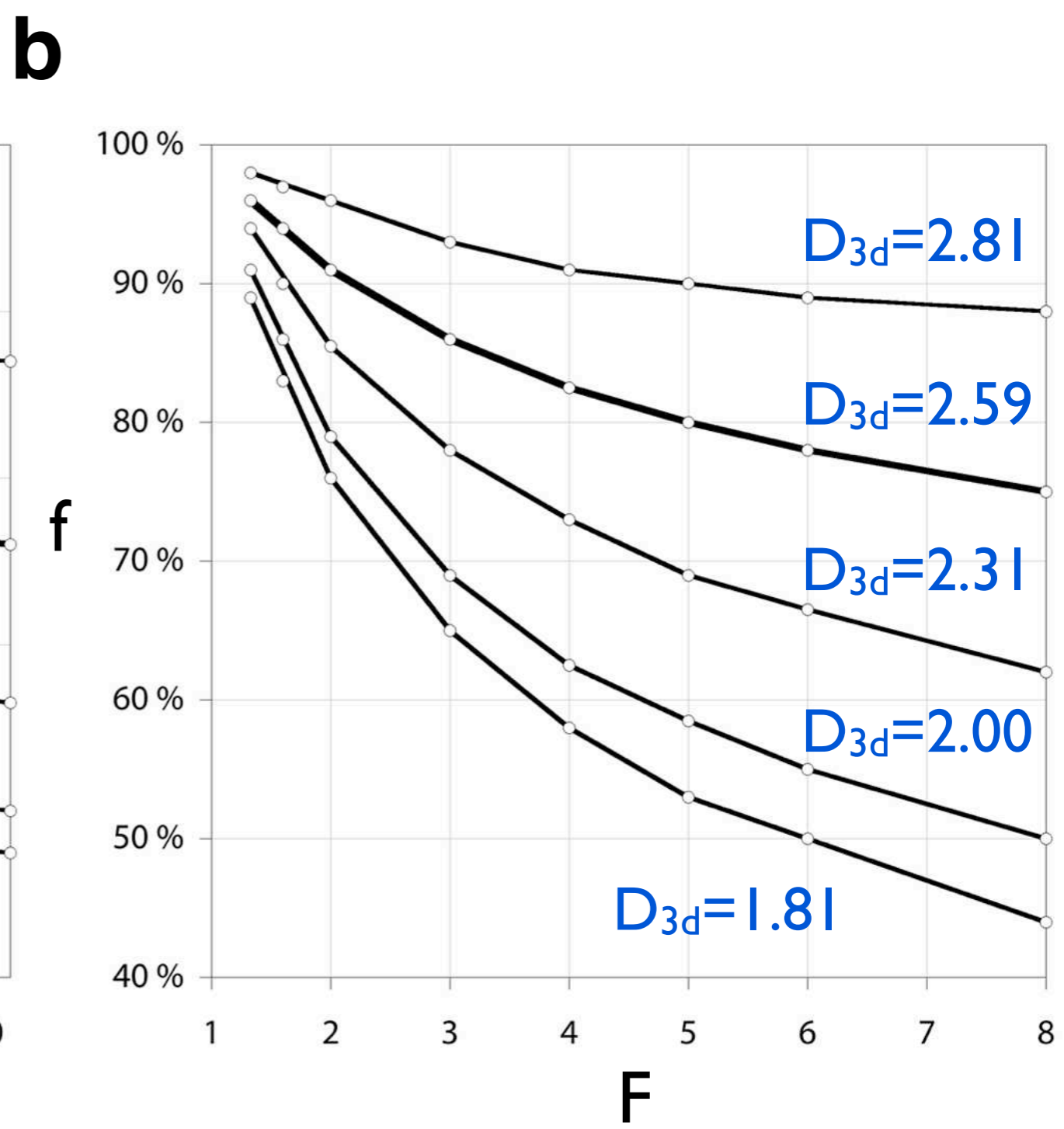
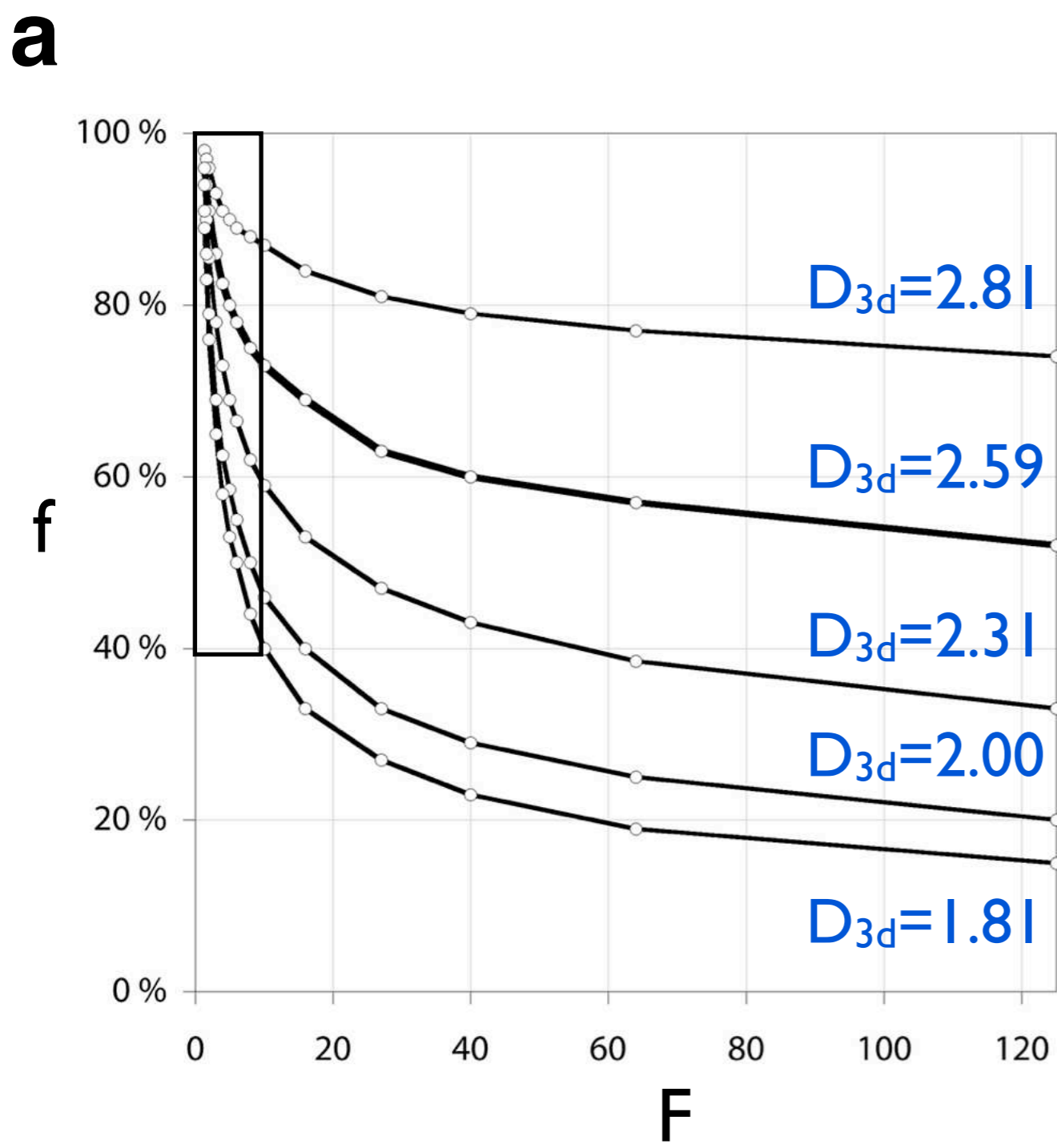
(b)  $D_{3d}$  for fragmentation numbers  $F \leq 8$ ;

(c)  $D_{2d}$  for fragmentation numbers  $F \geq 8$ , note that  $D_{2d} = D_{3d} - 1$ ;

(d)  $D_{2d}$  for fragmentation numbers  $F \leq 8$ , note that  $D_{2d} = D_{3d} - 1$ .

Plots (c) and (d) correspond to region framed in (a) and (b).





**Figure I3.9**

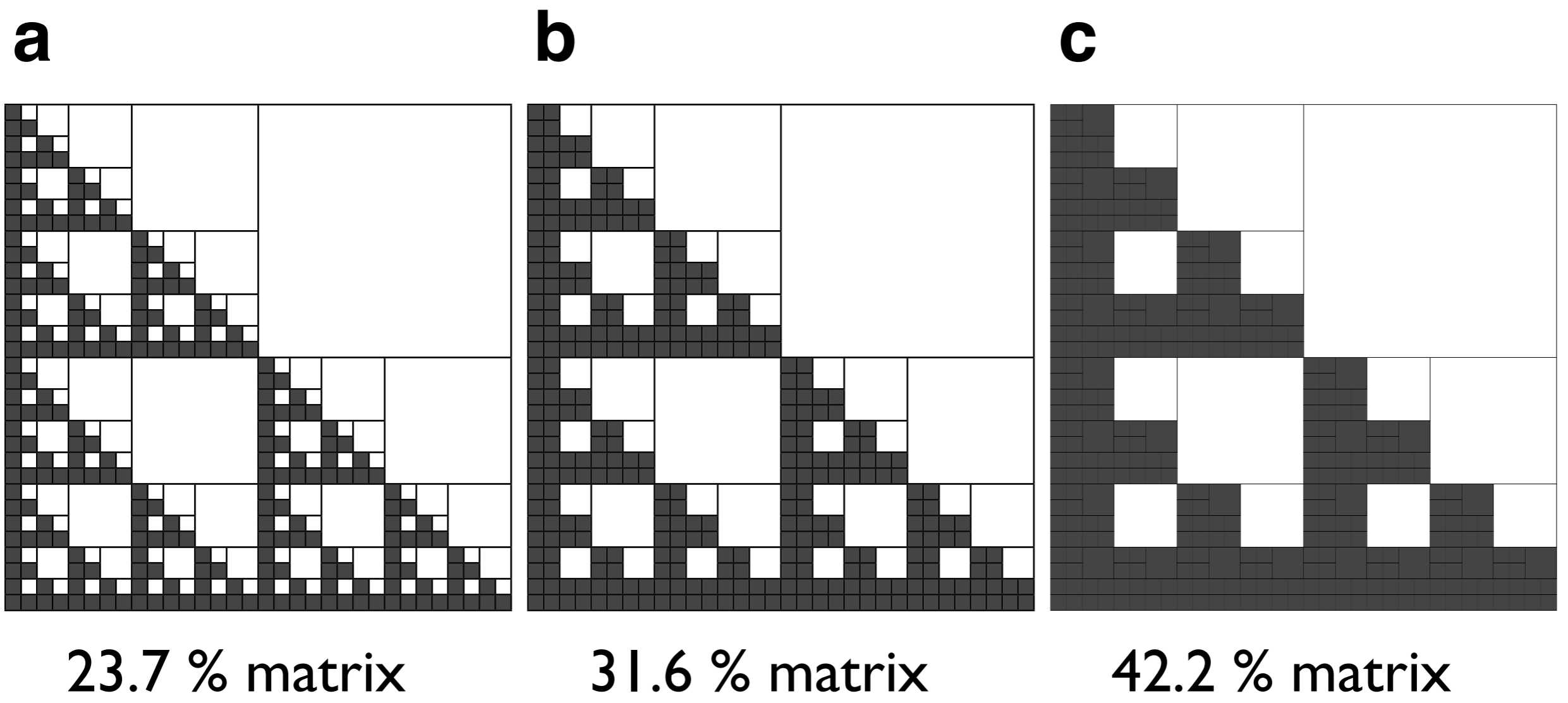
Fractal dimensions from different fragmentation processes.

Iso-lines of fractal dimension,  $D_{3d}$ , for varying fragmentation numbers,  $N$ , and fragmentation fractions,  $f$ .

(a) Curves of  $D_{3d}$  for ( $0\% \leq f \leq 100\%$ ) and ( $0 \leq F \leq 125$ );

(b) expanded view of plot indicated by rectangle in (a) for ( $50\% \leq f \leq 100\%$ ) and ( $0 \leq F \leq 8$ ).





**Figure 13.10**

Concept of fragmentation matrix.

Area fraction of fragmentation matrix is defined by matrix length scale,  $L_m$ ;

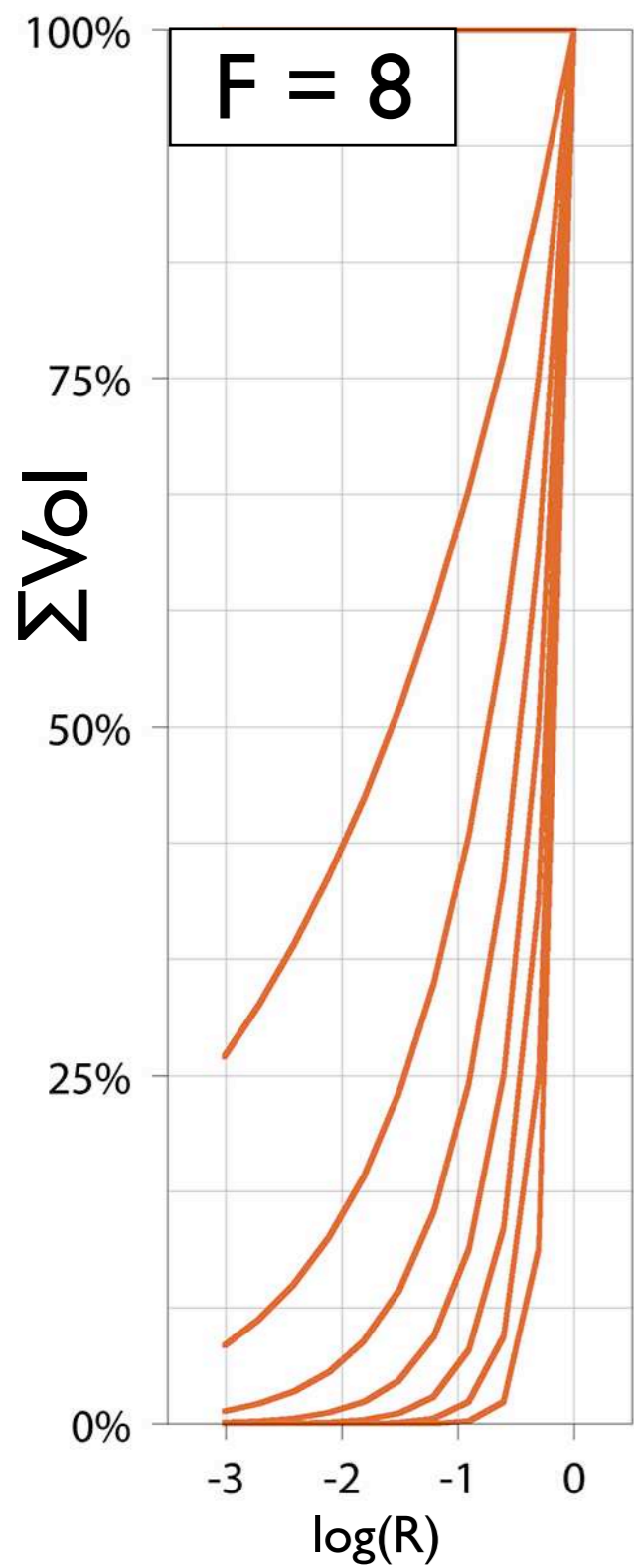
$L_m = g / G$  where  $g$  = largest grain in matrix and  $G$  = largest grain present in image.

(a)  $L_m = 1 / 32$  and matrix content = 23.7 %;

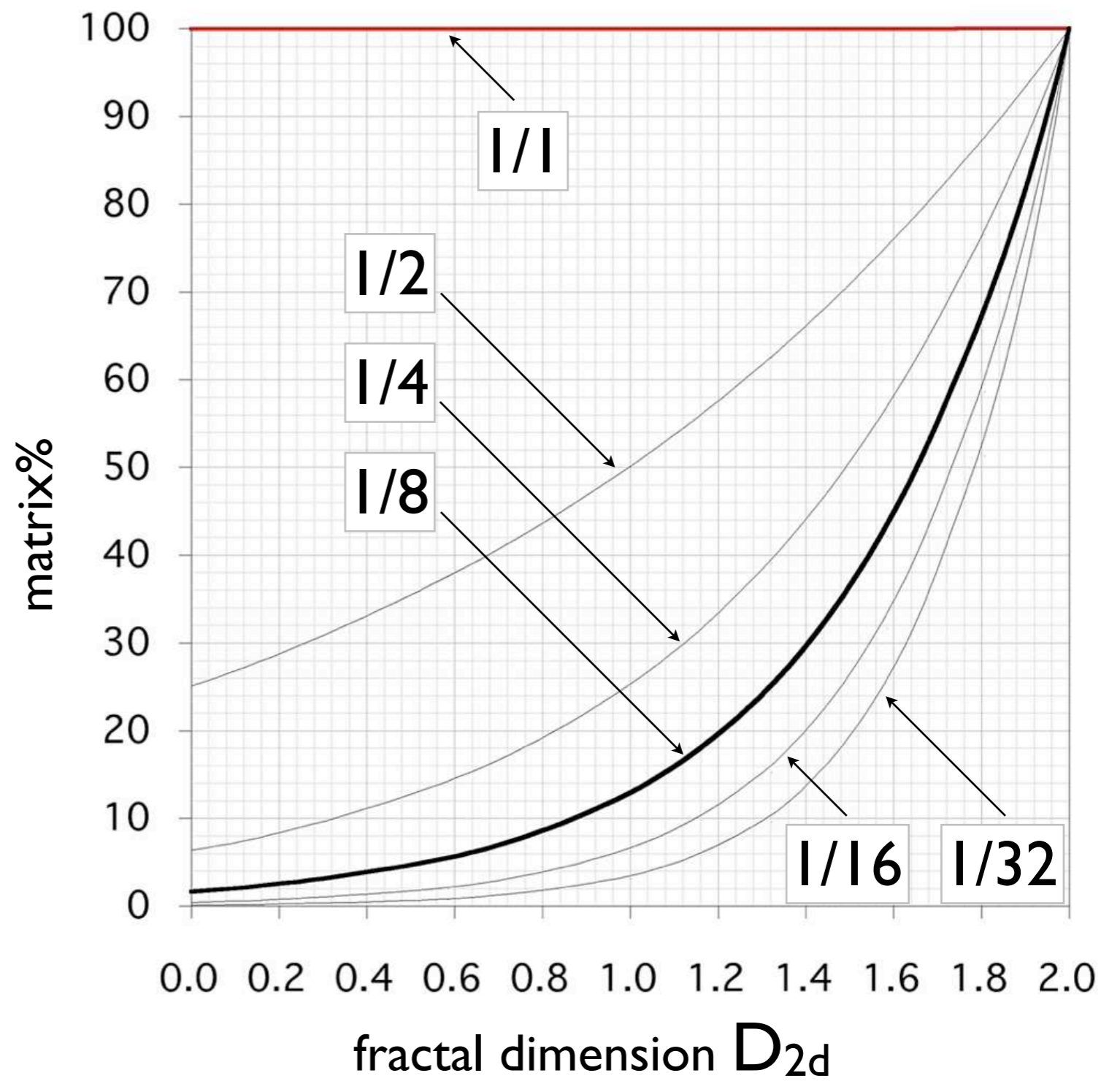
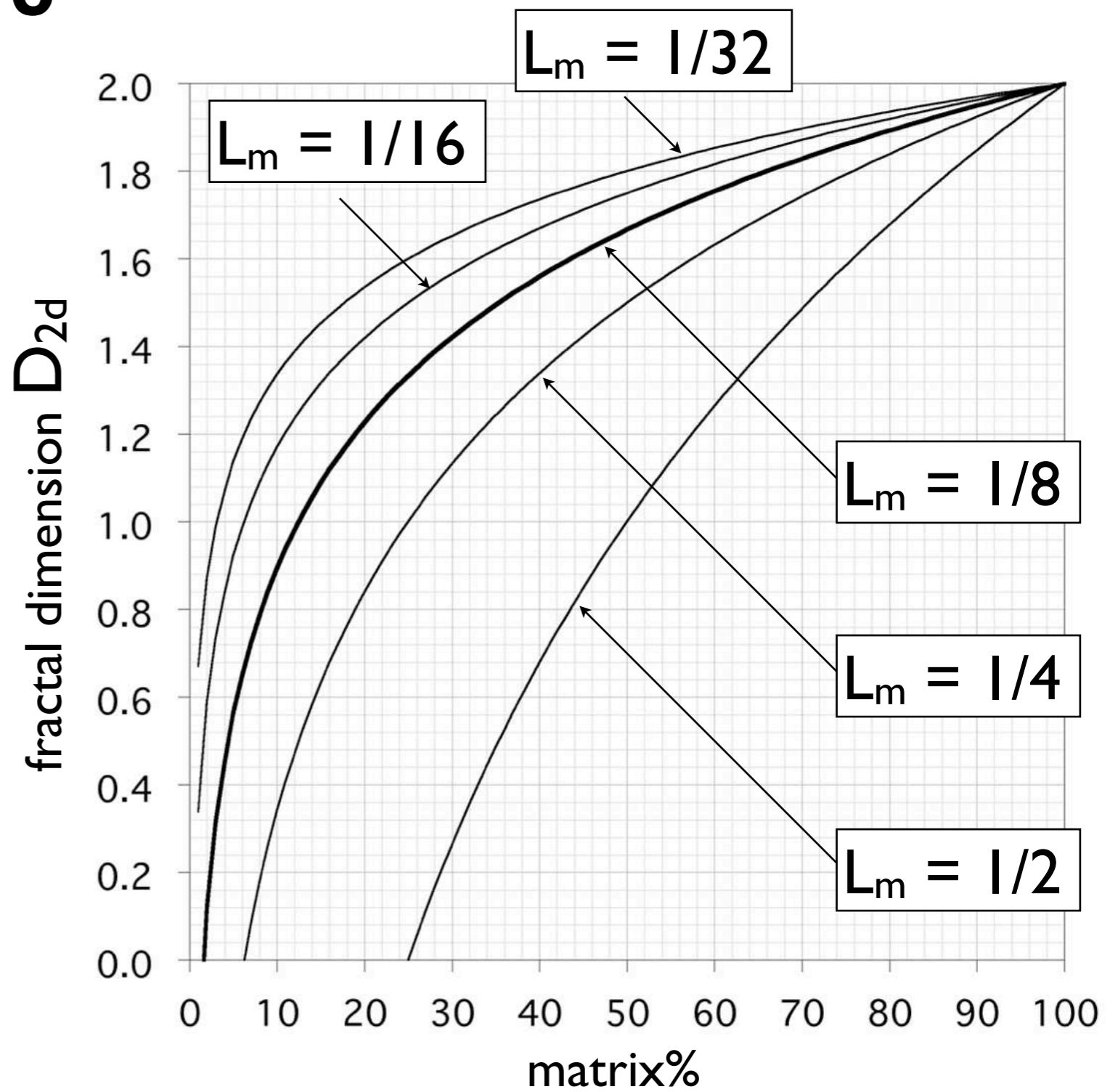
(b)  $L_m = 1 / 16$  and matrix content = 31.6 %;

(c)  $L_m = 1 / 8$  and matrix content = 42.2 %.



**a**

$f$	$D_{3d}$
8/8	3.00
7/8	2.81
6/8	2.59
5/8	2.32
4/8	2.00
3/8	1.59
2/8	1.00
1/8	0.00

**b****c****Figure 13.11**

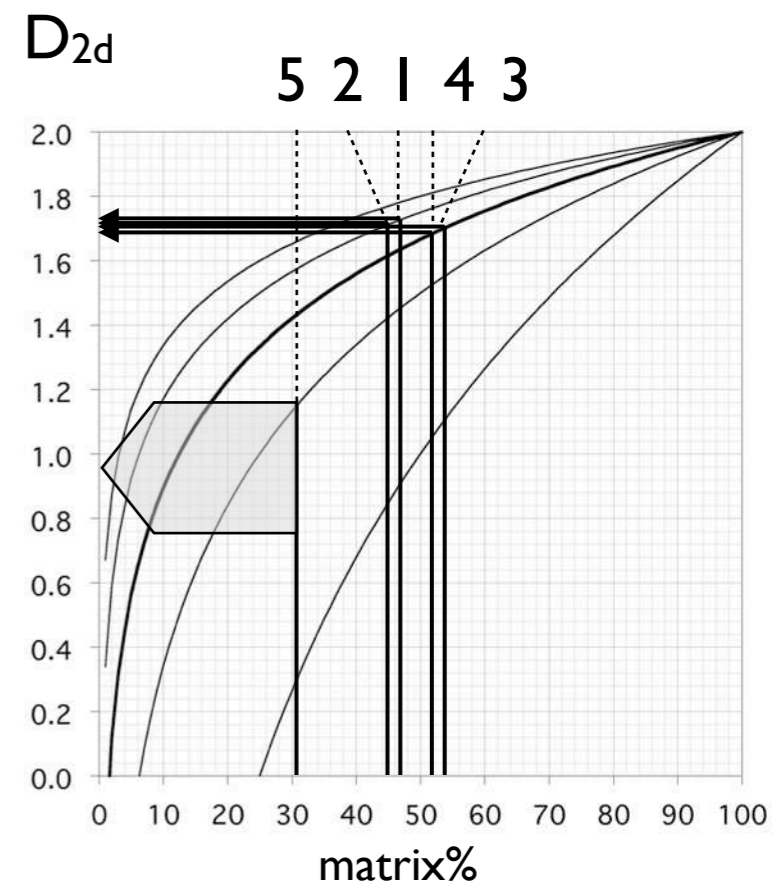
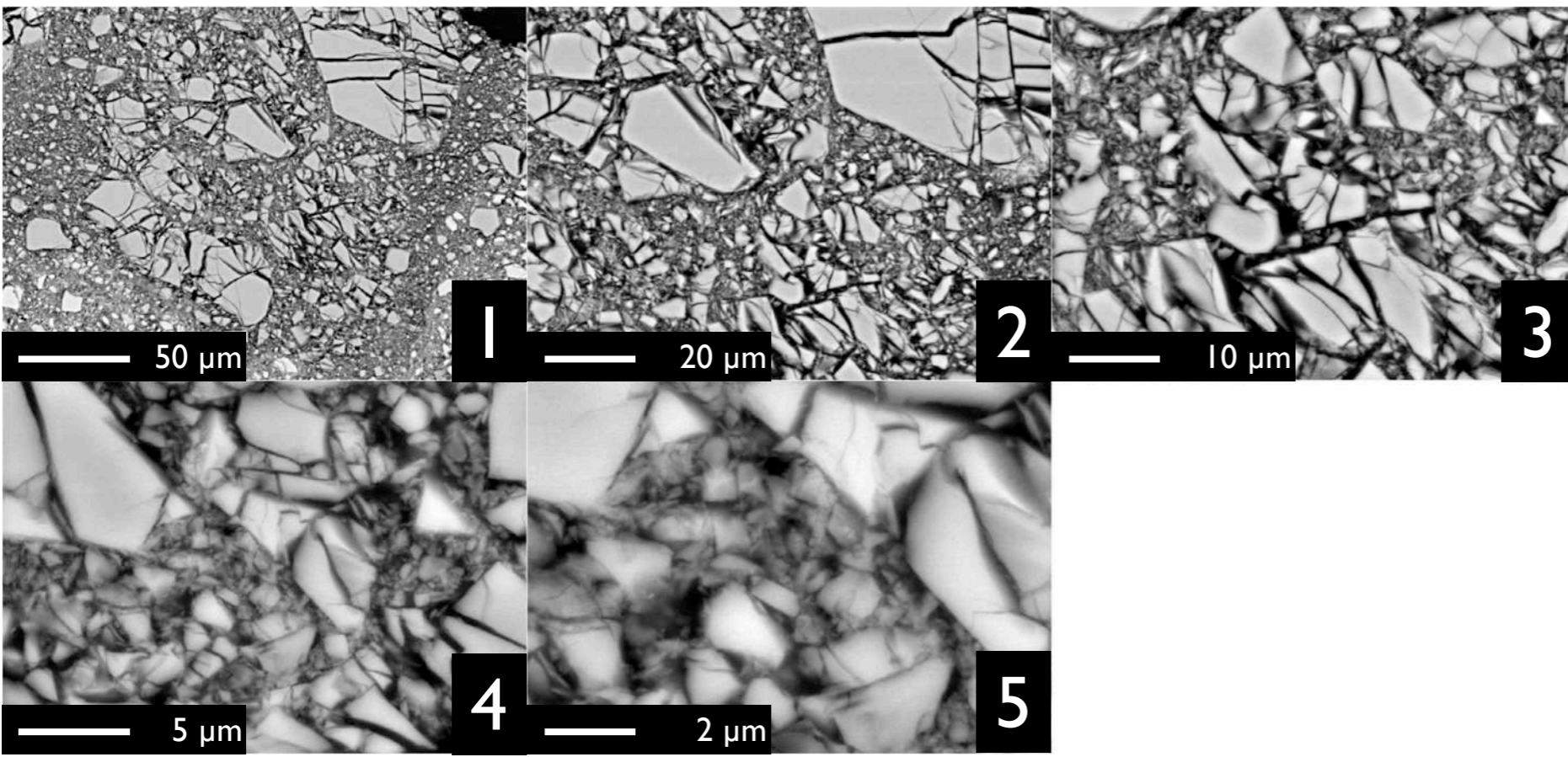
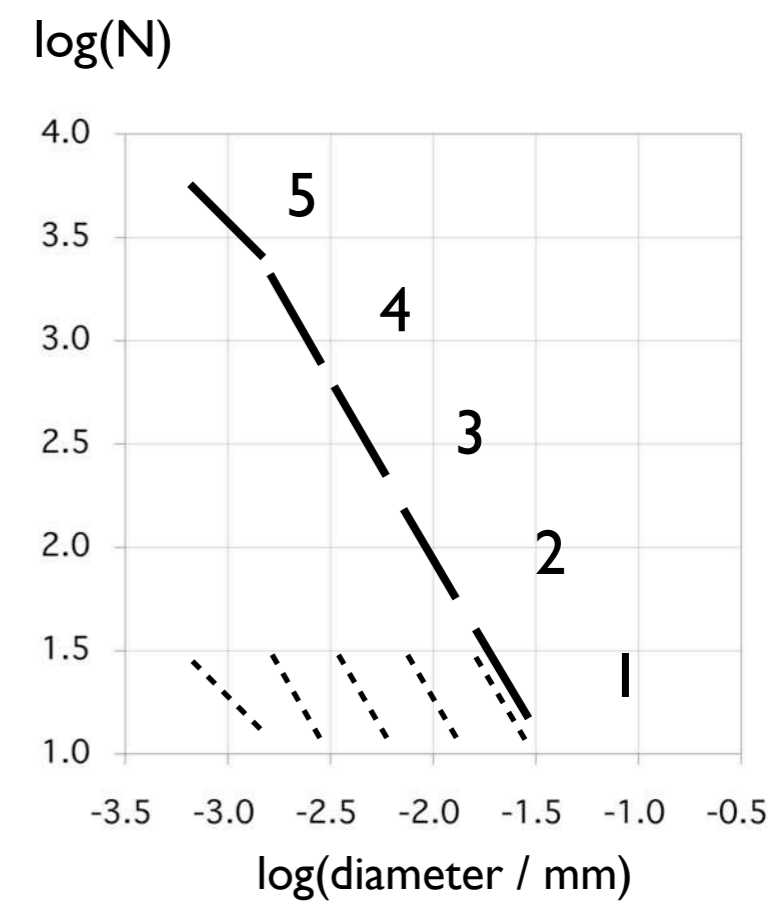
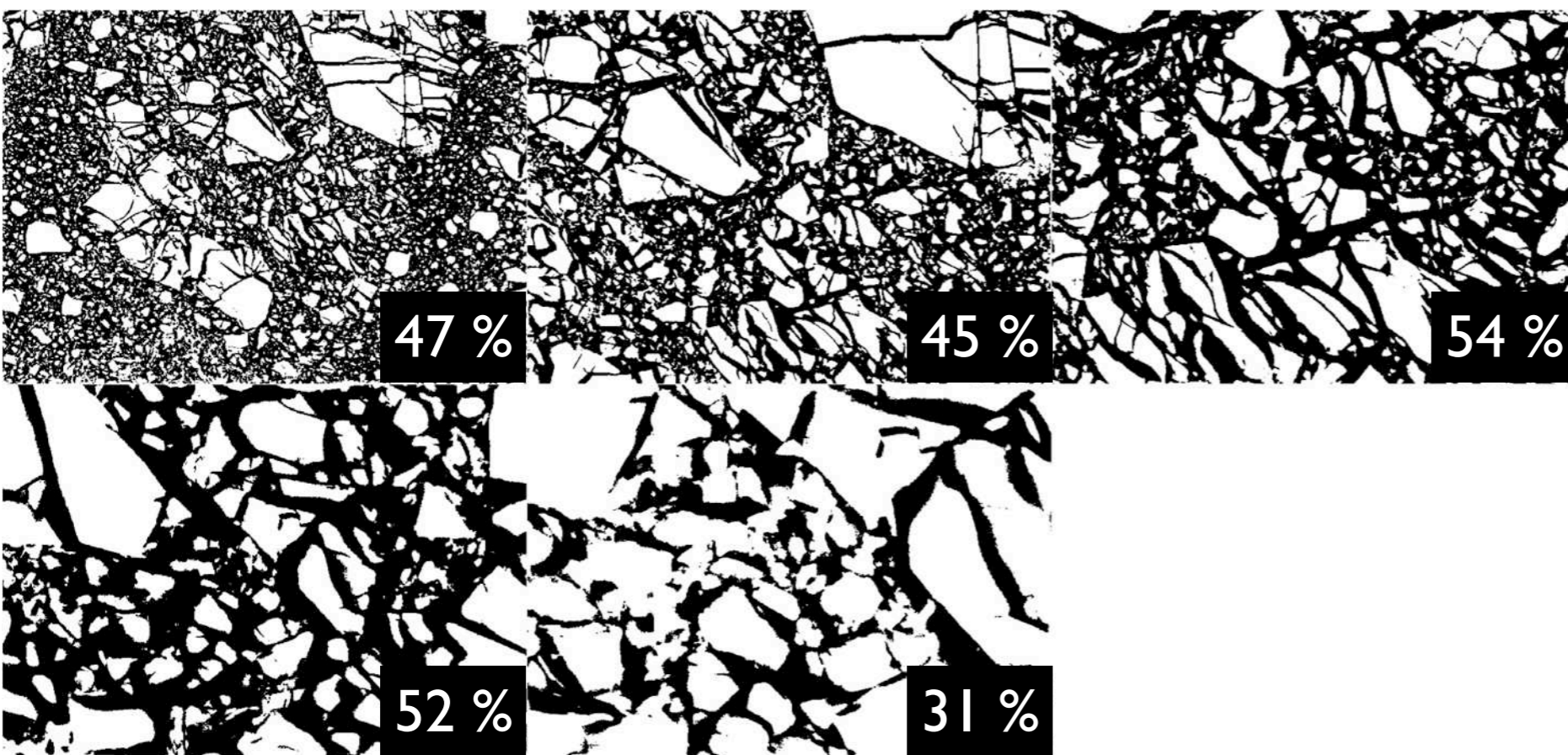
Apparent matrix content and fractal dimension.

(a) Cumulative volume,  $\Sigma V$ , is shown as function of  $\log(R)$  for different  $D_{3d}$  (compare Figure 13.4.c); fragmentation number  $F = 8$ ,  $f$  = fragmentation fraction,  $R$  = size of fragments,  $D_{3d}$  = fractal dimension of the size distribution of 3-D grains;

(b) matrix contents,  $m\%$ , is shown as functions of  $D_{2d}$  for different matrix lengths scales,  $L_m$ ; the fractal dimension ( $0.00 \leq D_{2d} \leq 2.00$ );

(c) matrix-D diagram showing fractal dimension ( $0.00 \leq D_{2d} \leq 2.00$ ) versus matrix content for a range of matrix lengths scales,  $L_m = 1/32, 1/16, 1/8, 1/4$  and  $1/2$  (for  $L_m$  see Figure 13.10).



**a****b****c****d****Figure 13.12**

Deriving the fractal dimension from the matrix content.

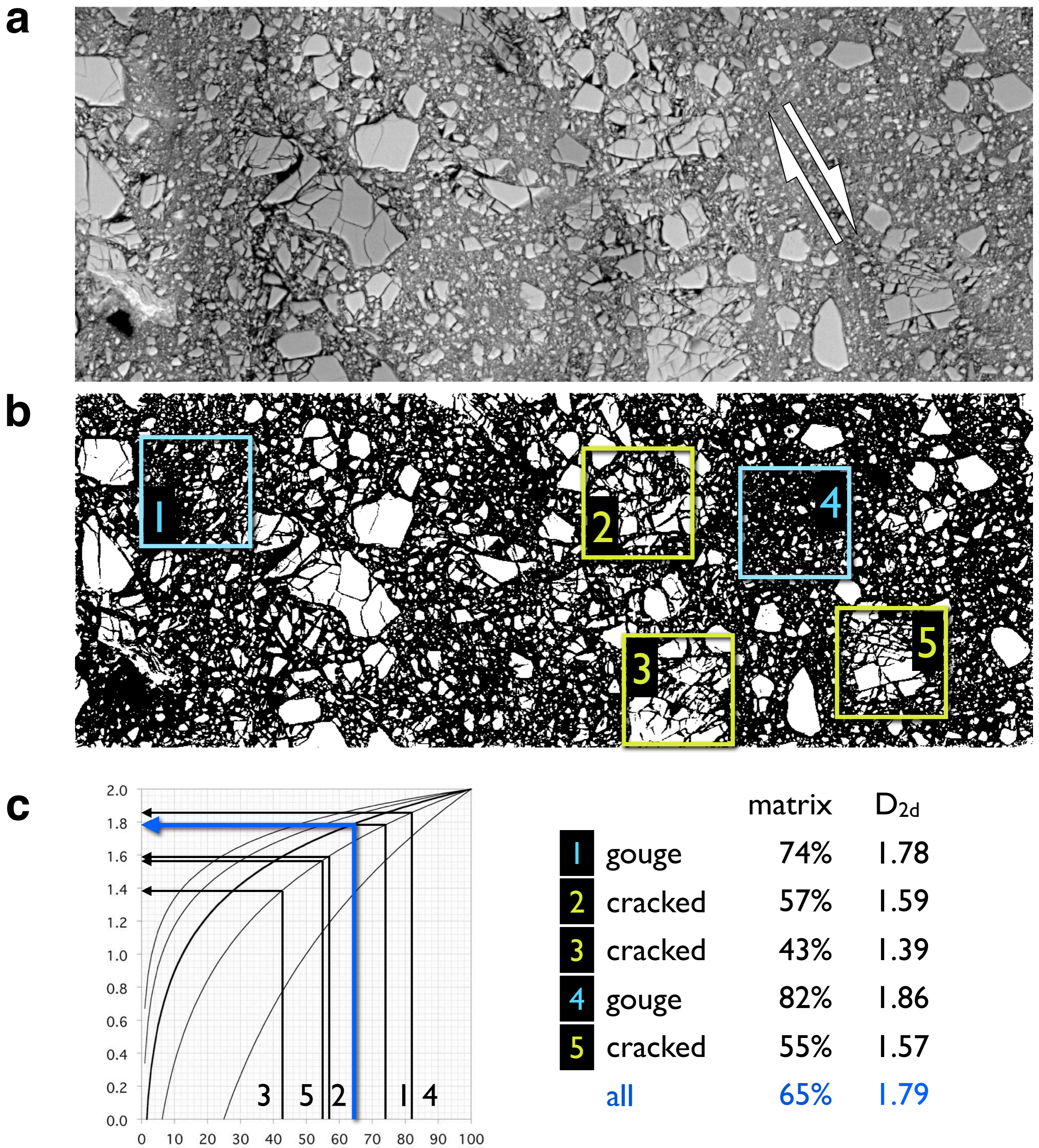
(a) SEM micrograph of fault rock at increasing magnification;

(b) bitmaps of (a), area percentage of matrix indicated;

(c) matrix-D diagram (see Figure 13.11) with matrix data for samples 1 to 5; matrix length scale,  $L_m = 1/16$  for 1,2,  $L_m = 1/8$  for 3,4, and  $L_m > 1/4$  for 5;

(d)  $\log(N)$ - $\log(R)$  plot constructed from D-values (slopes) of areas 1 to 5.





**Figure 13.13**

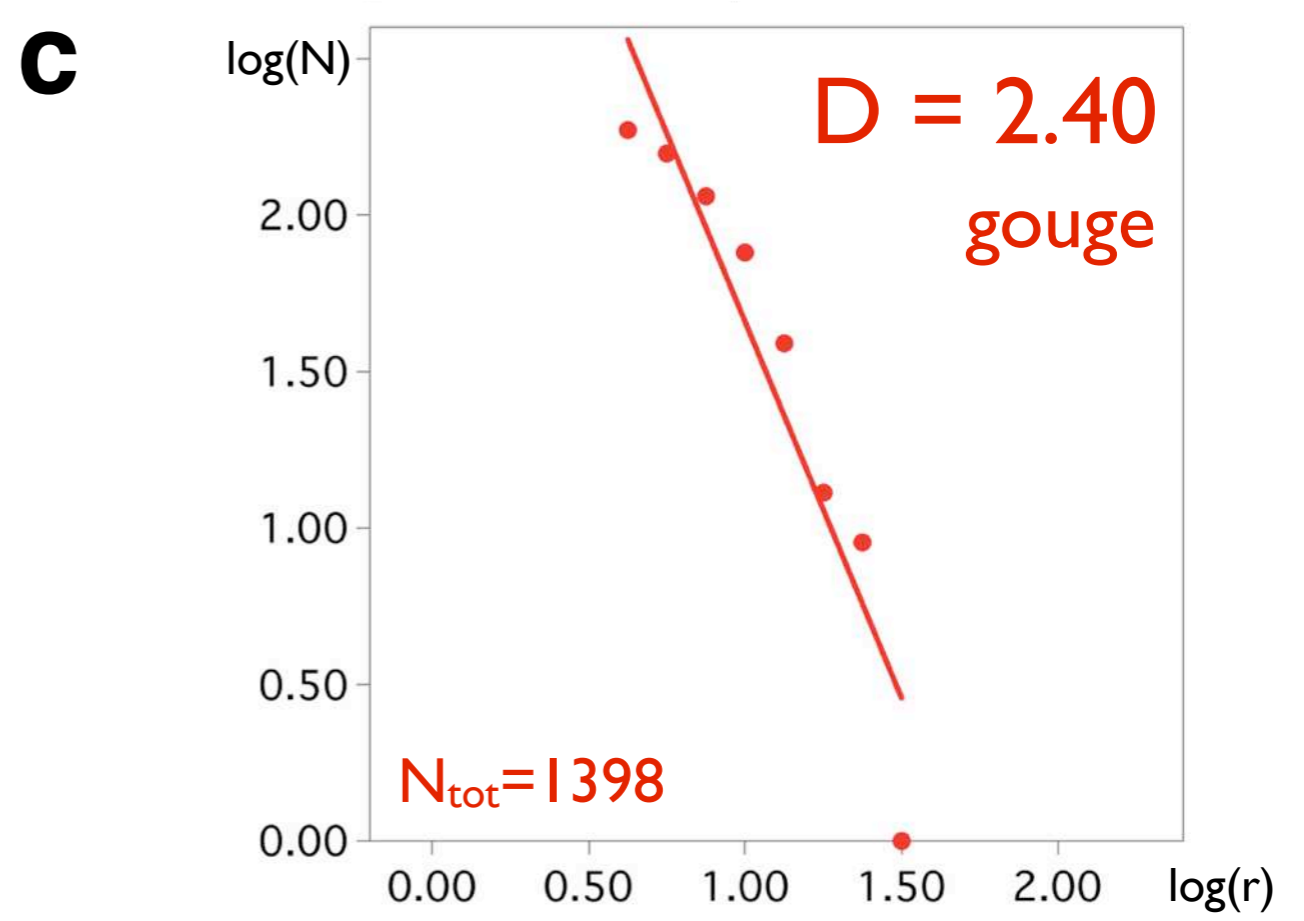
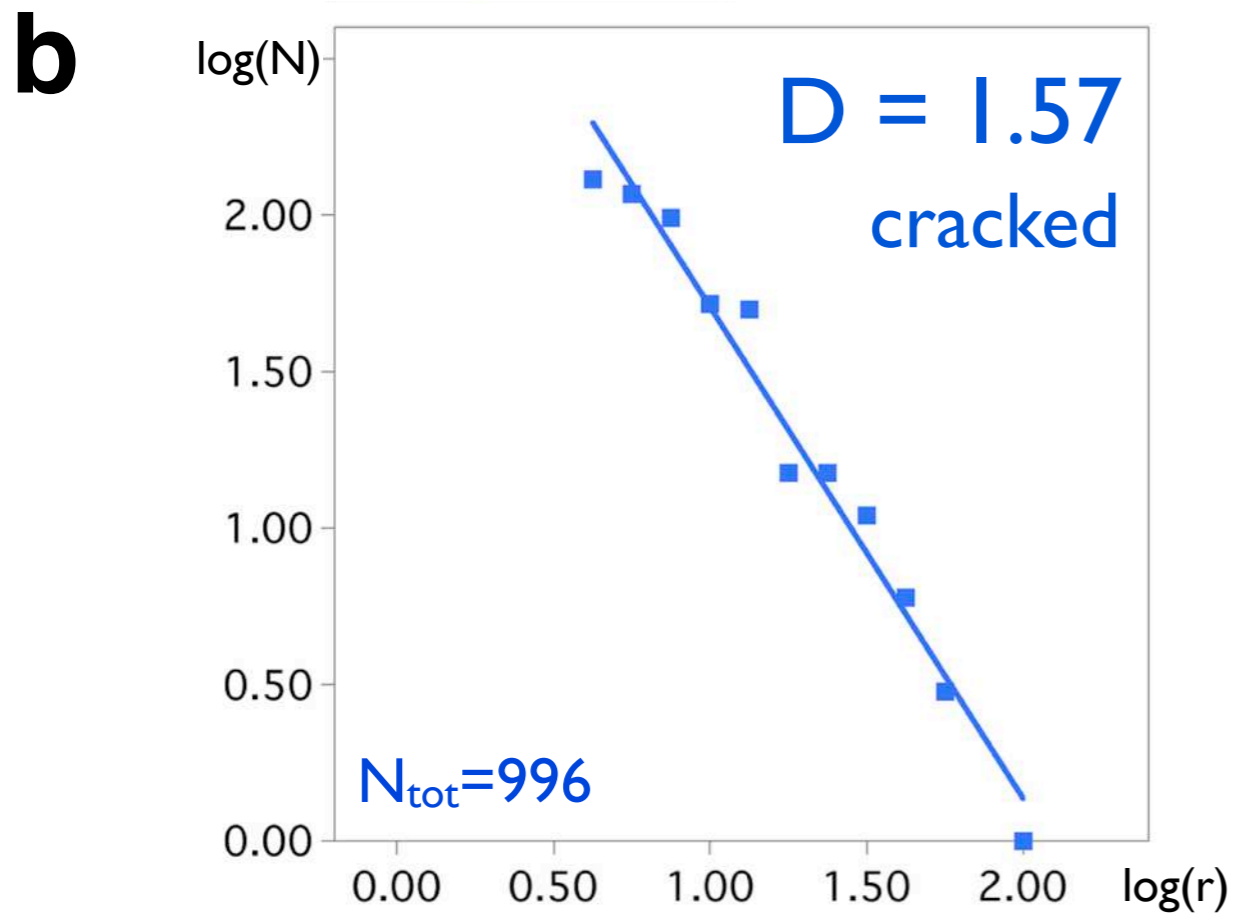
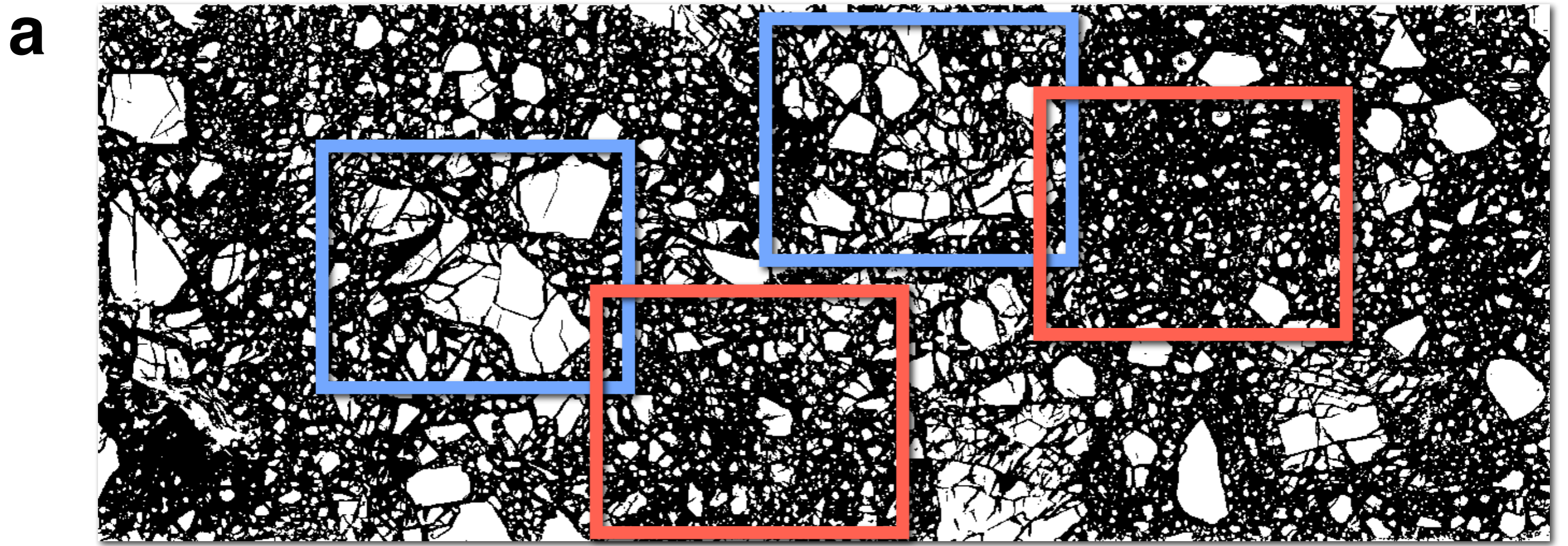
Local variation of the fractal dimension.

(a) SEM micrograph of experimentally produced cataclastic shear zone (displacement vertical, shear sense indicated, sample courtesy Nynke Keulen);

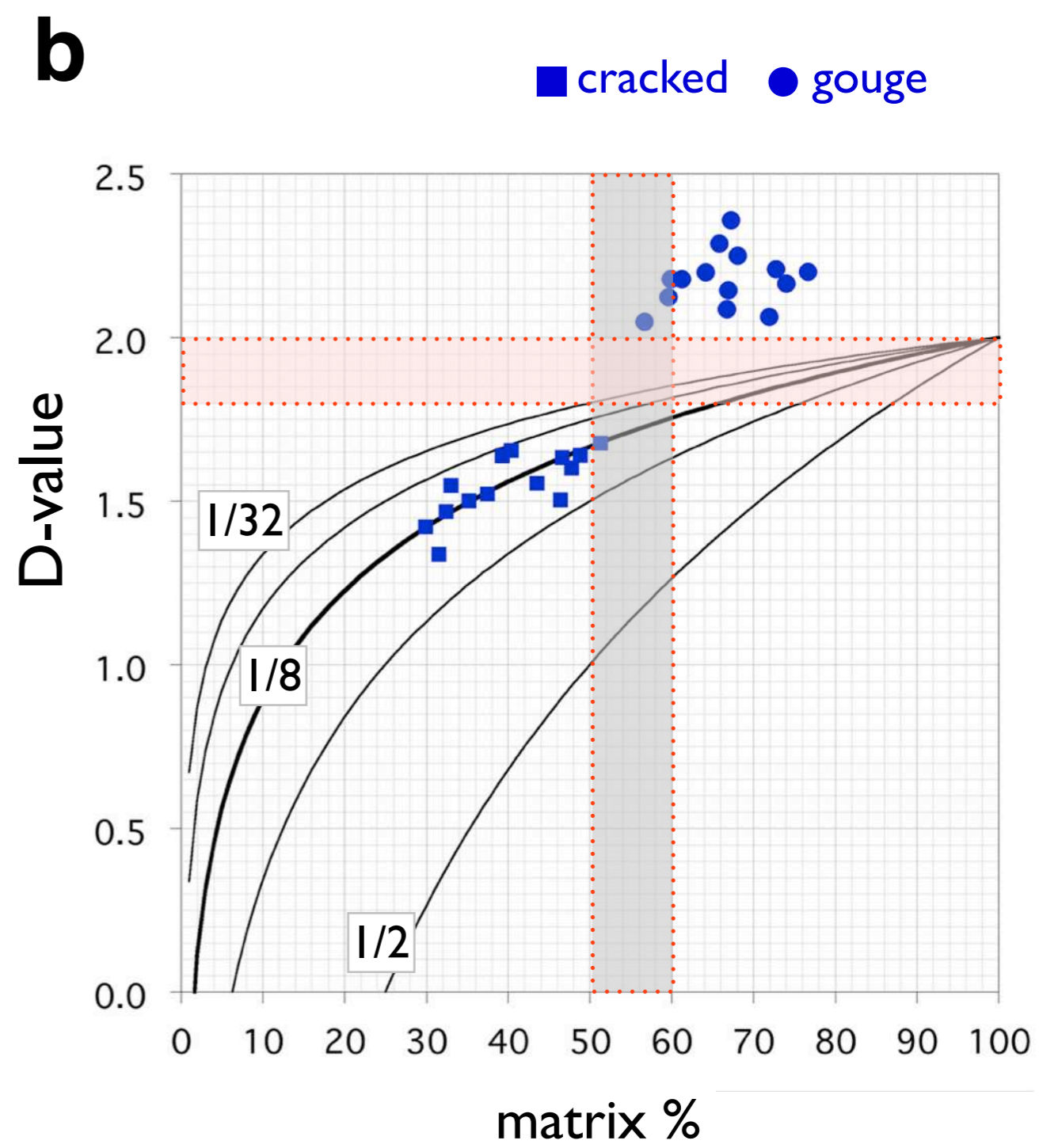
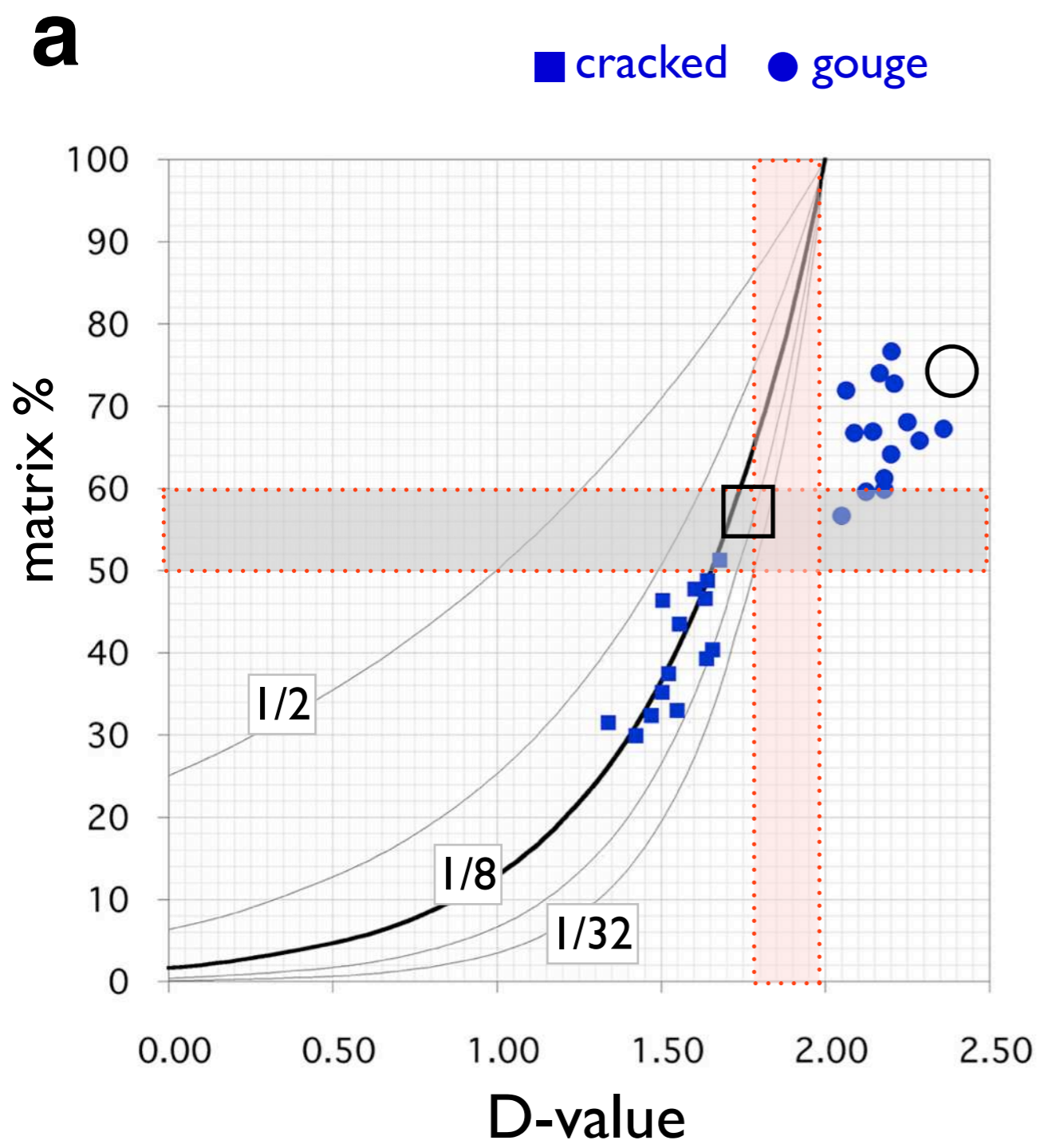
(b) bitmap of (a), 5 sites are indicated;

(c) matrix-D diagram (see Figure 13.11) with matrix data for sites 1 to 5; blue = data for entire bitmap; matrix length scale,  $L_m = 1/8$  for entire image,  $L_m = 1/4$  for areas 1 to 5.





**Figure 13.14**  
 Non-fractal size distribution.  
 (a) Bitmap of experimentally produced cataclastic shear zone (Figure 13.13);  
 (b) grain size analysis of fragmented part;  
 (c) grain size analysis of part with mature gouge;  
 $r$  = size,  $N$  = number of fragments, total number indicated;  $D$  = negative slope of line fit.



**Figure 13.15**

Fractal dimension from measured matrix content and D-value from measured grain size analysis.

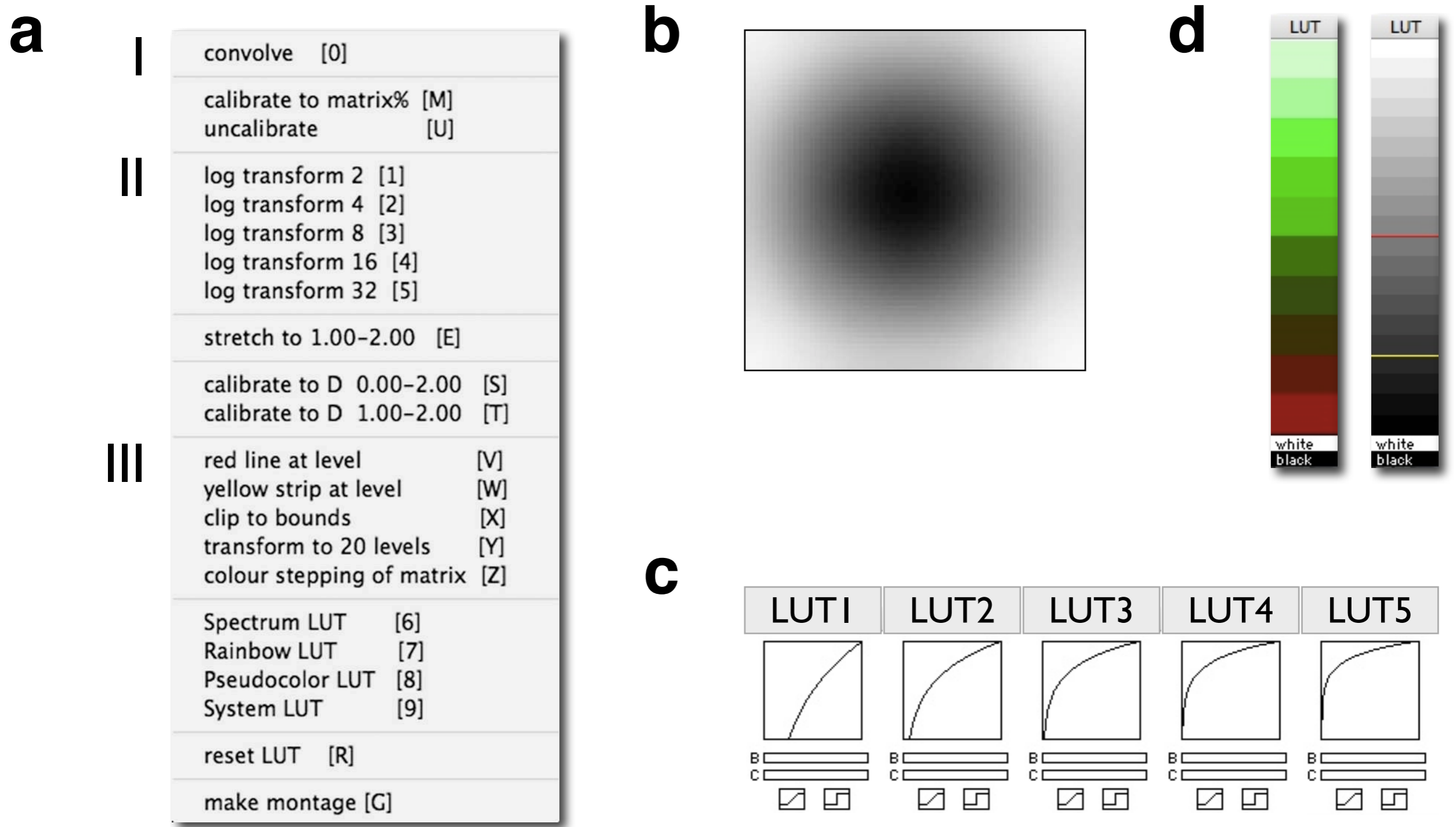
(a) Matrix contents as functions of D, expanded version of plot shown in Figure 13.11.b; black symbols are measured data points for cracked and gouge material;

(b) expanded matrix-D diagram (see Figure 13.11.c); same data points as in Figure 13.15.a; note  $D > 2$  for gouge.

Lines denote matrix lengths scales  $L_m = 1/32, 1/16, 1/8, 1/4$  and  $1/2$  (see Figure 13.11.c);

pink and gray shaded areas outline range of matrix%- and D- values separating cracked from gouge material.





**Figure 13.16**

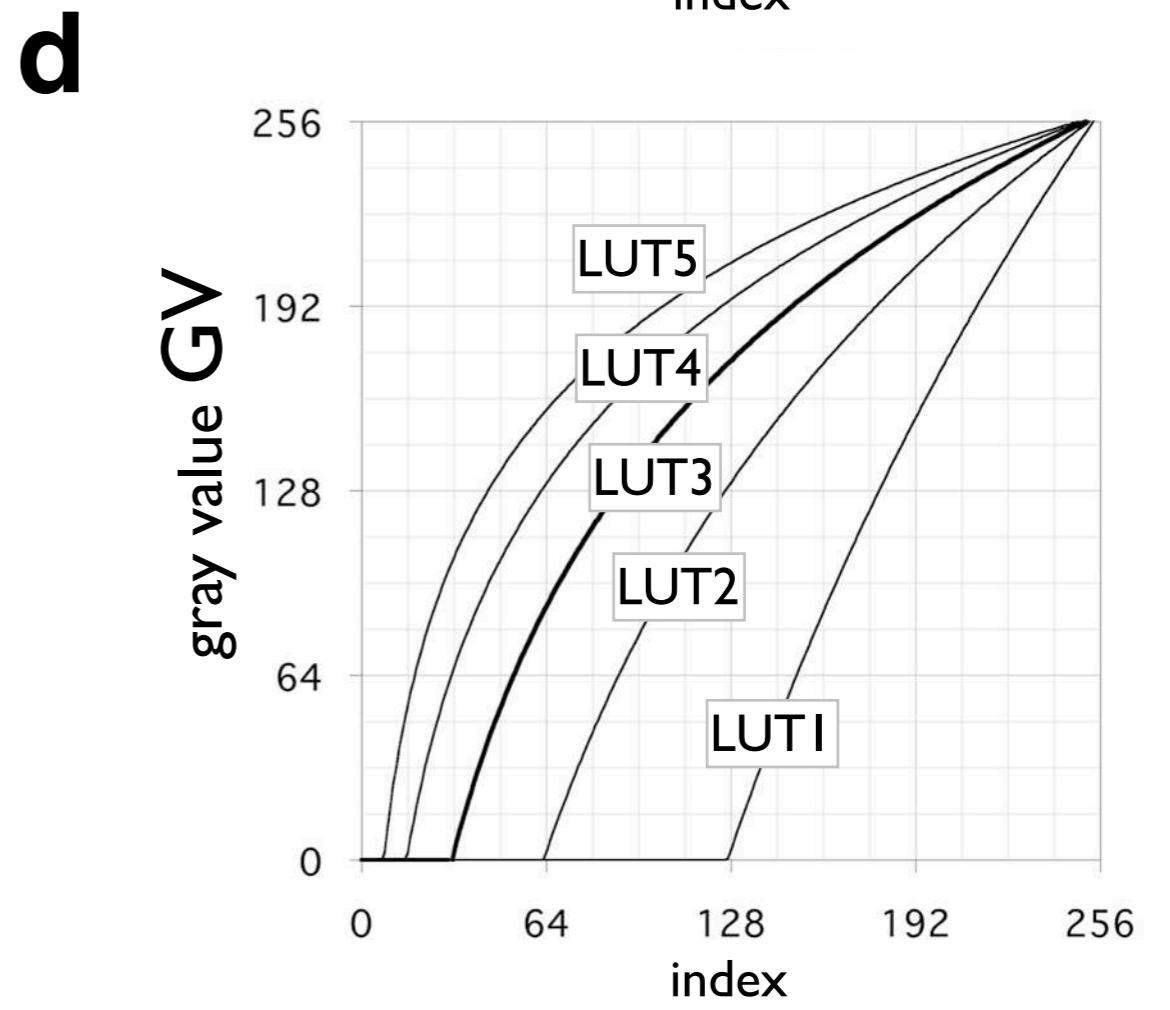
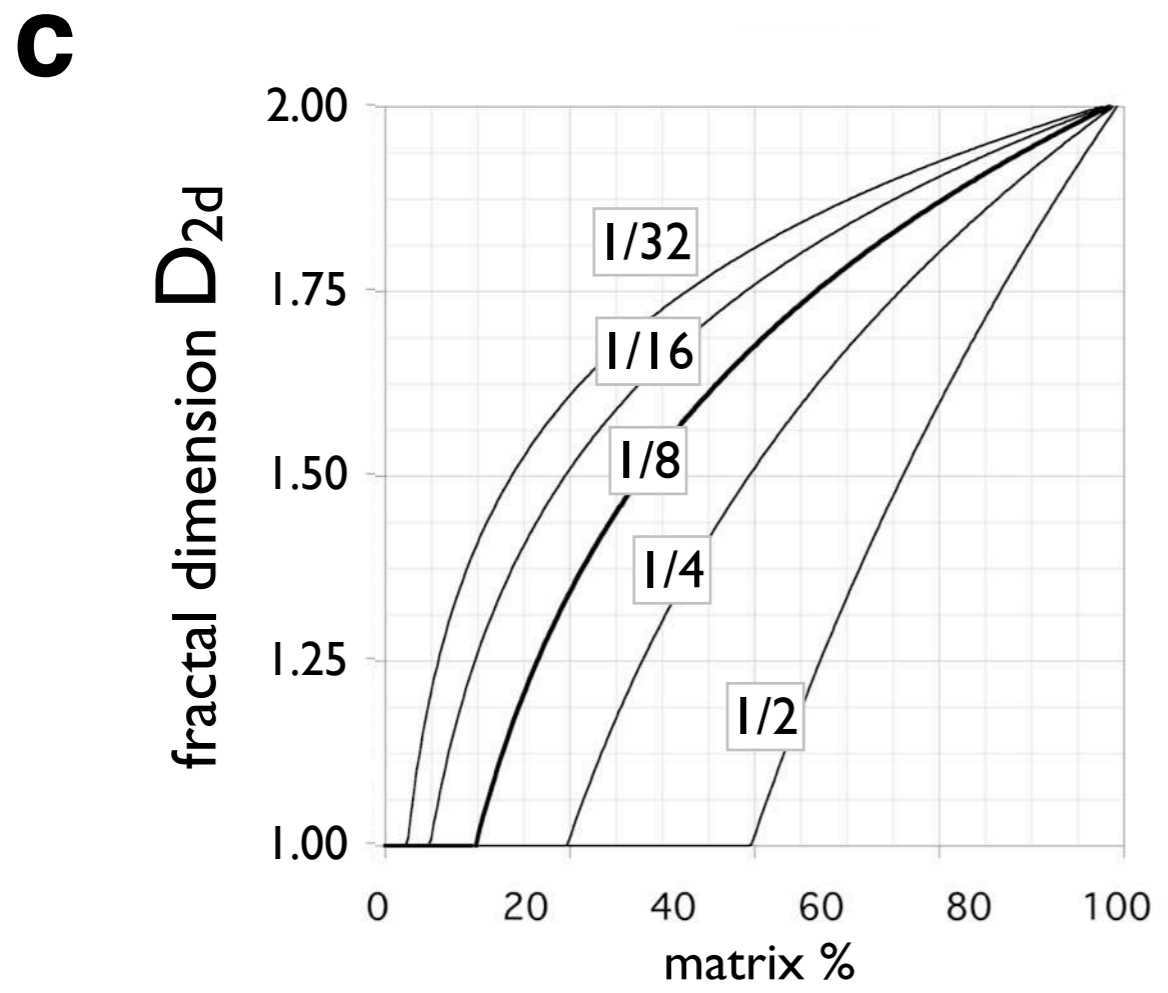
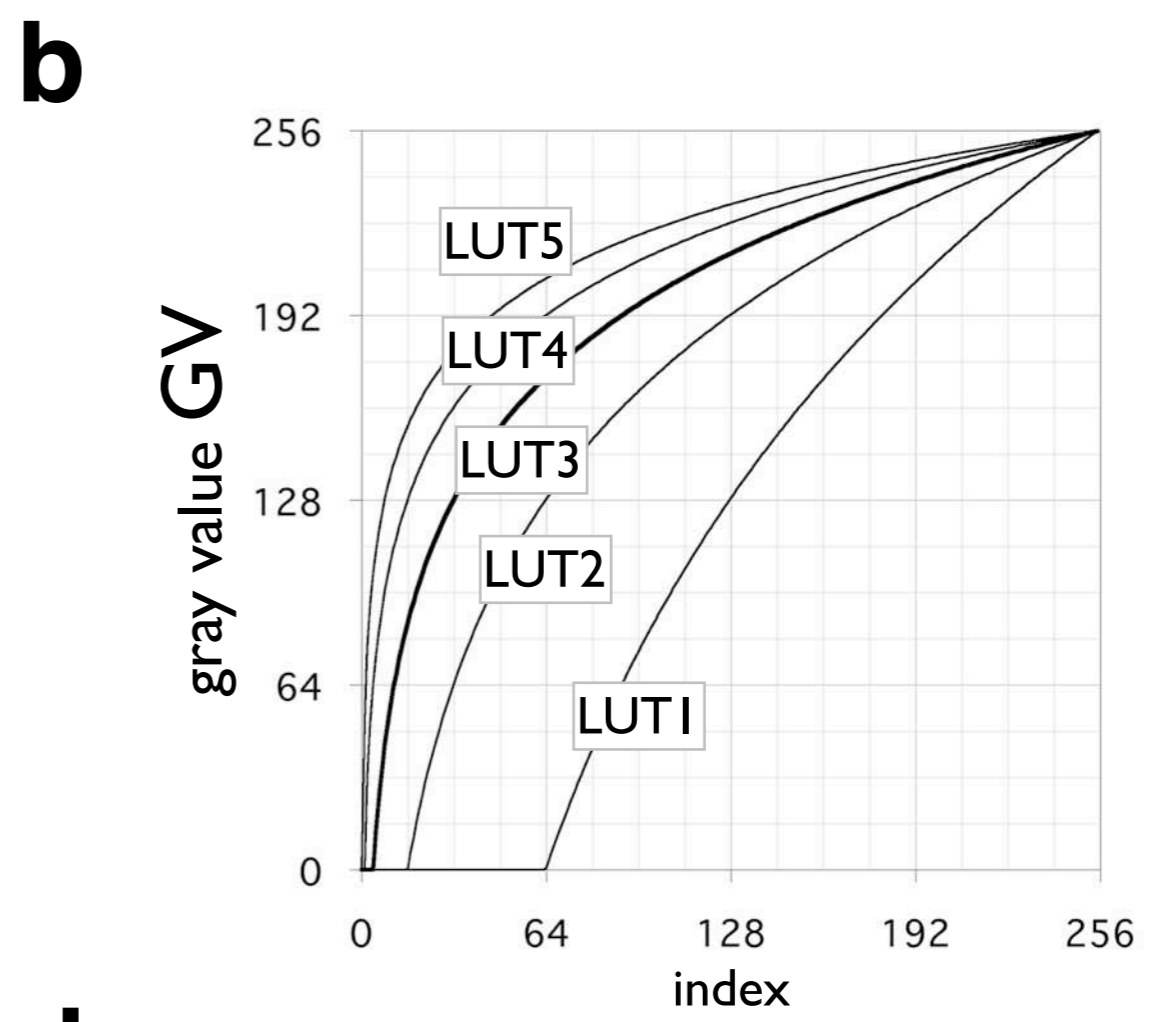
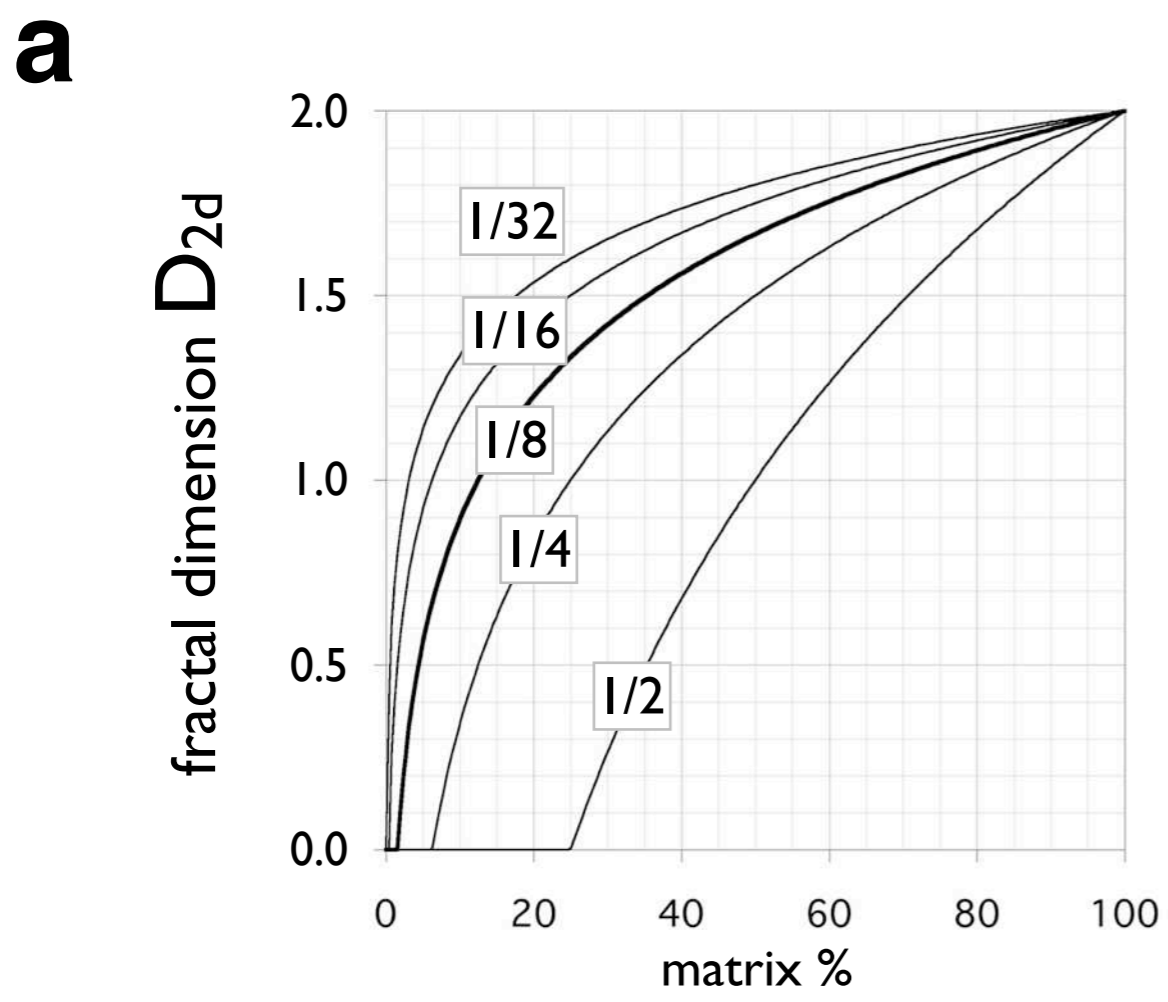
Mapping fractal dimensions of grain size distributions.

(a) The Lazy D map menu;

(b) step I: creating matrix density maps:  $63 \times 63$  Gauss filter kernel shown as brightness image;

(c) step II: converting to fractal dimension: look-up tables LUT1 to LUT5 for matrix length scales,  $L_m$ , from  $1/2$  to  $1/32$

(d) step III: using color LUTs to visualize (example from left to right: 10 colors for 0-100% matrix, 20 steps with red line at 50% and yellow strip at 80%).



**Figure 13.17**

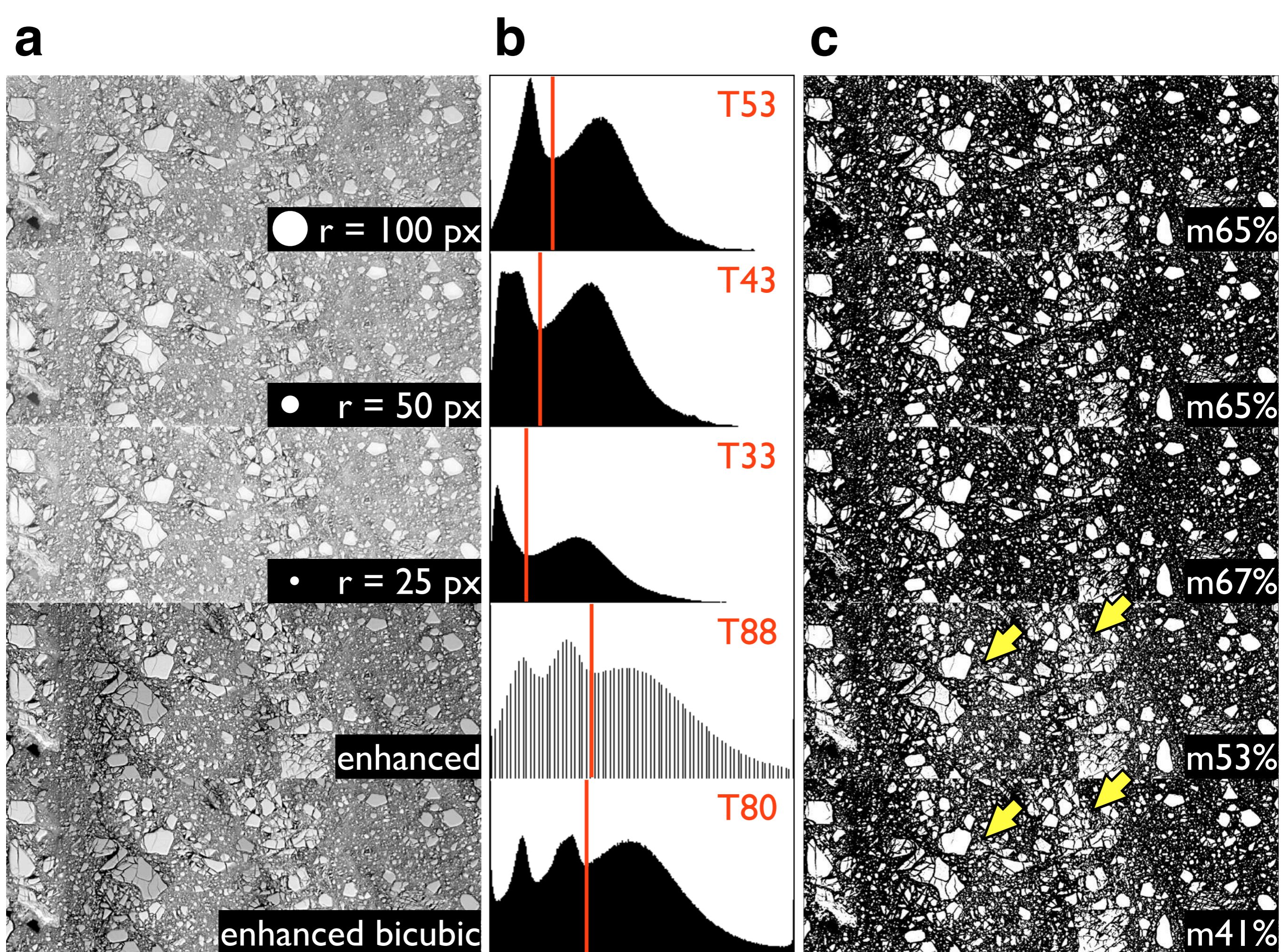
Creating look-up tables from matrix-D diagrams.

Matrix density (0% to 100%) is converted to index (0 to 255), fractal dimension,  $D_{2d}$ , to gray value, GV, (0 to 255).

(a) and (c) for ( $0.00 \leq D_{2d} \leq 2.00$ ); matrix lengths scales,  $L_m$ , are indicated.

(b) and (d) for ( $1.00 \leq D_{2d} \leq 2.00$ ); name of LUT indicated.





**Figure 13.18**

Preparing the bitmap.

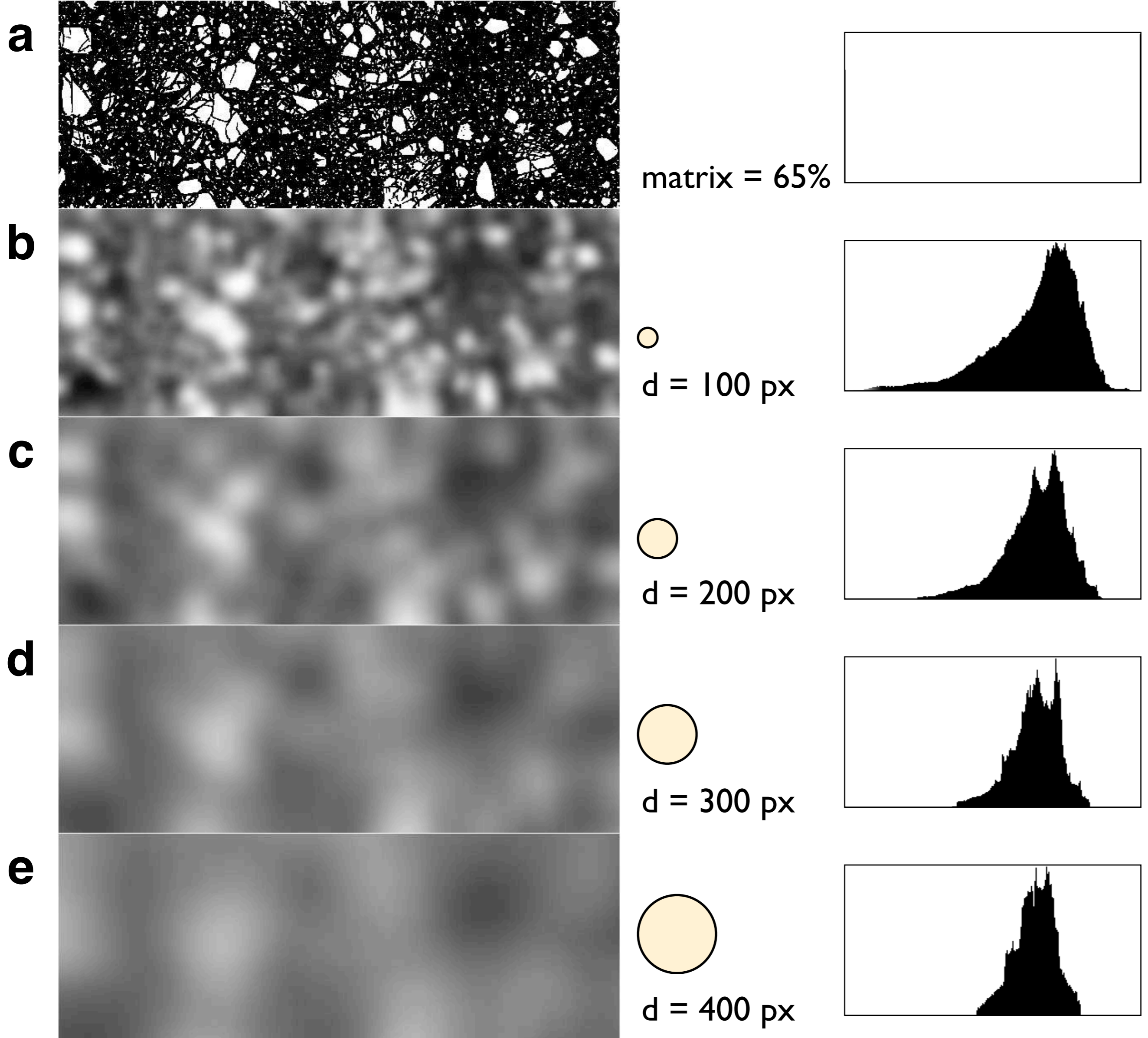
Pre-processing by rolling ball background subtraction with decreasing radius, by enhancing, and bicubic smoothing (top to bottom).

(a) Grayscale images, radius of rolling ball indicated;

(b) histogram with threshold level indicated;

(c) resulting bitmap, average area fraction of matrix indicated; arrows point to areas where matrix is underestimated.





**Figure 13.19**

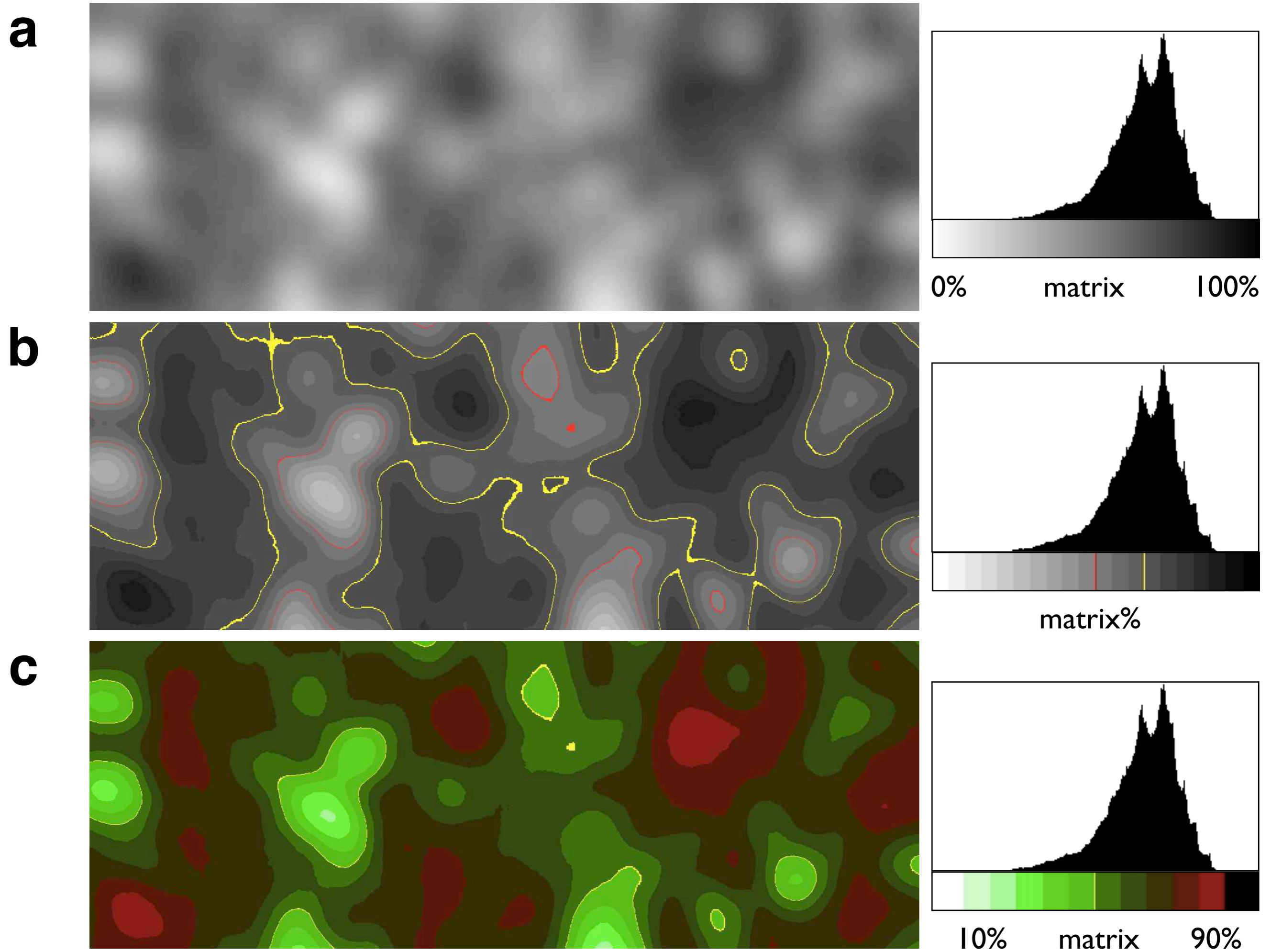
Creating matrix density maps by Gauss convolution..

Matrix density images are calculated by de-magnifying image proportional to  $63 \cdot 63$  Gauss filter kernel (maximum filter size for Image SXM), (2) convolving, and (3) re-magnifying to original size.

(a) Original bitmap, average gray value = 164, corresponding to matrix density of 65 %;

(b) to (e) matrix density images, the corresponding filter diameters are indicated; note that range in histograms decreases while average matrix density remains constant (65 %).





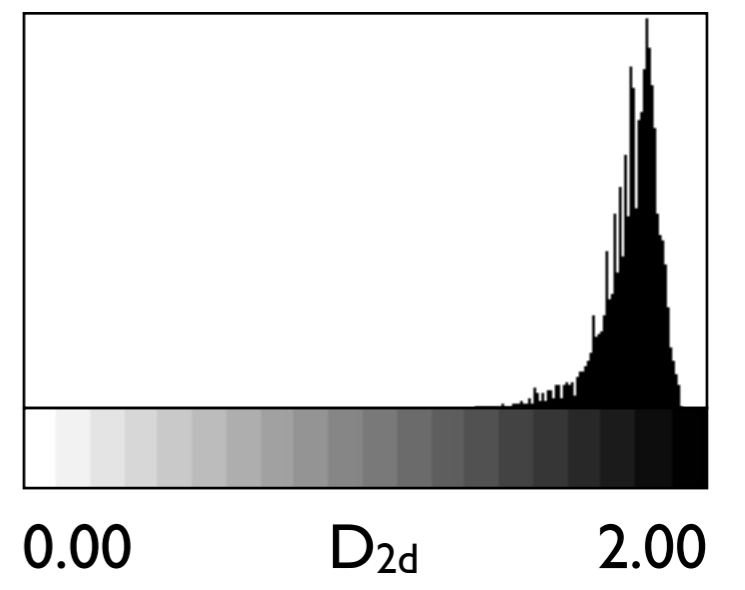
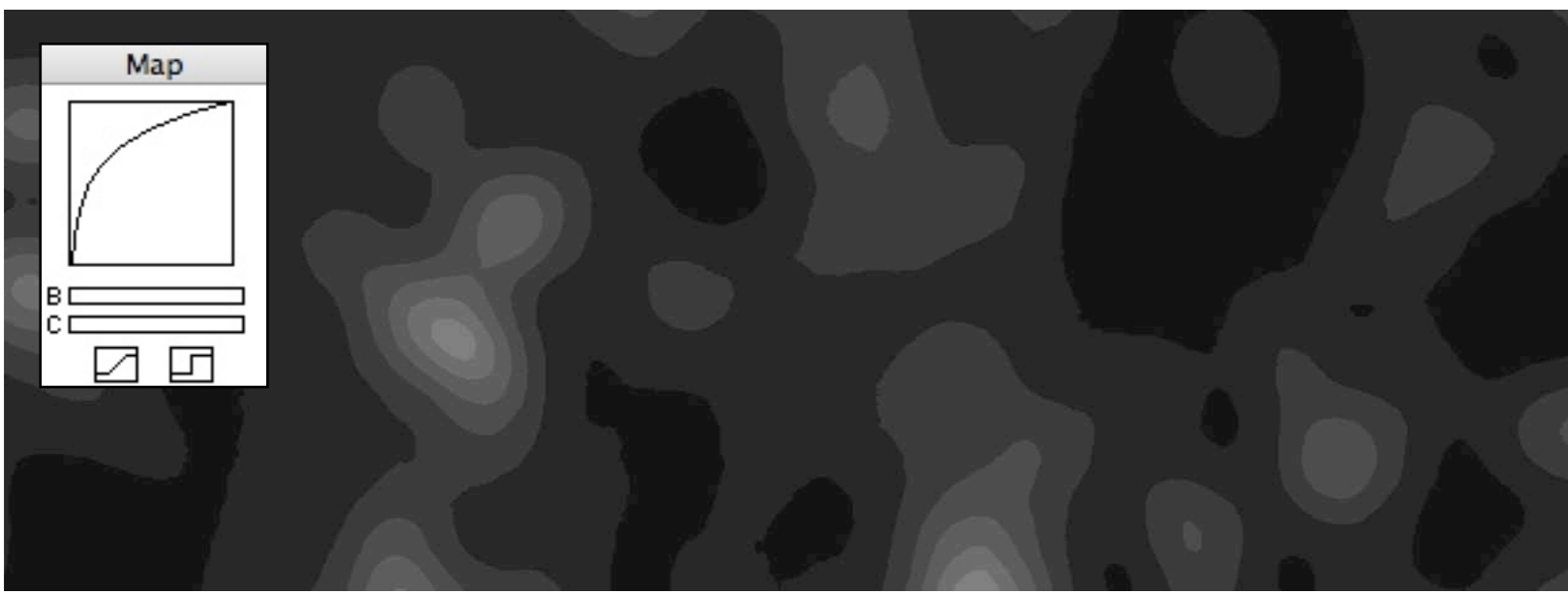
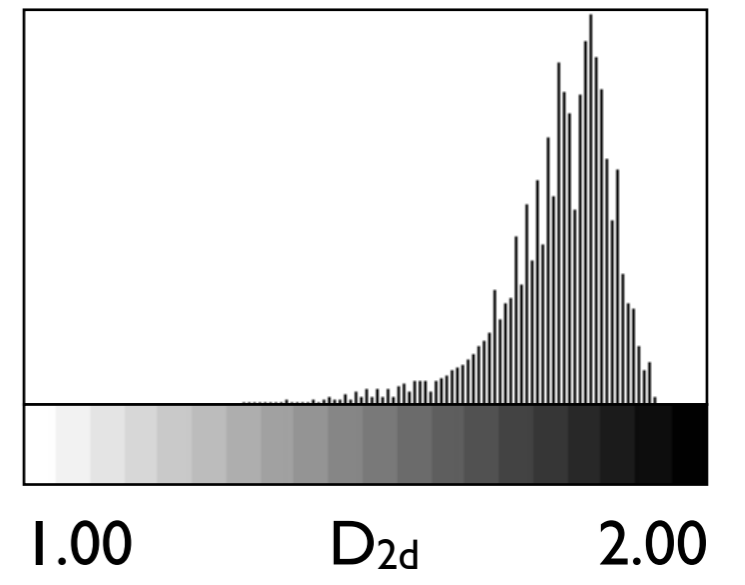
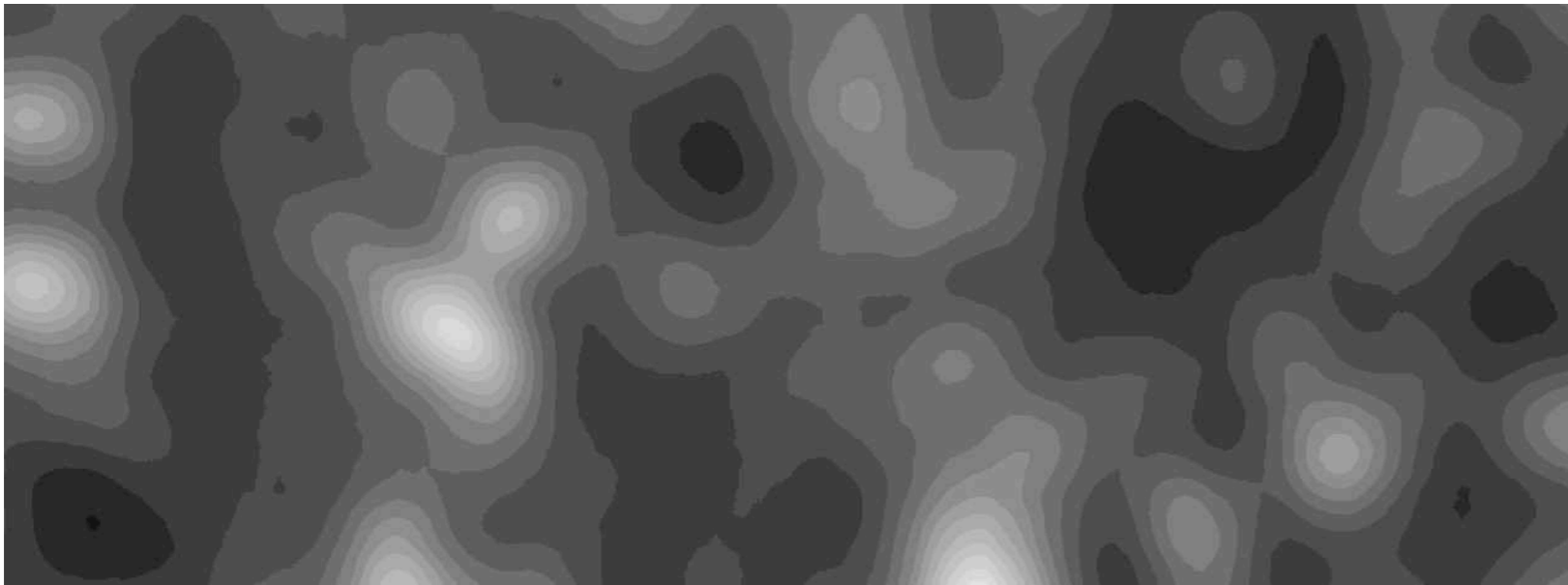
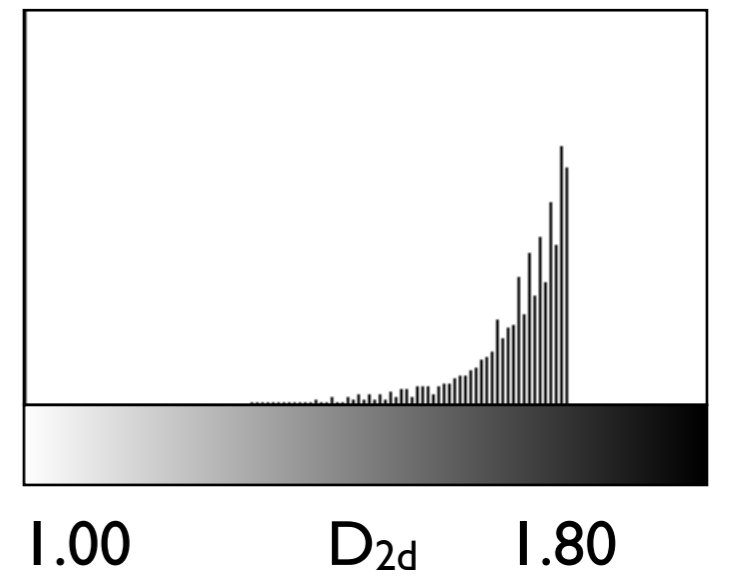
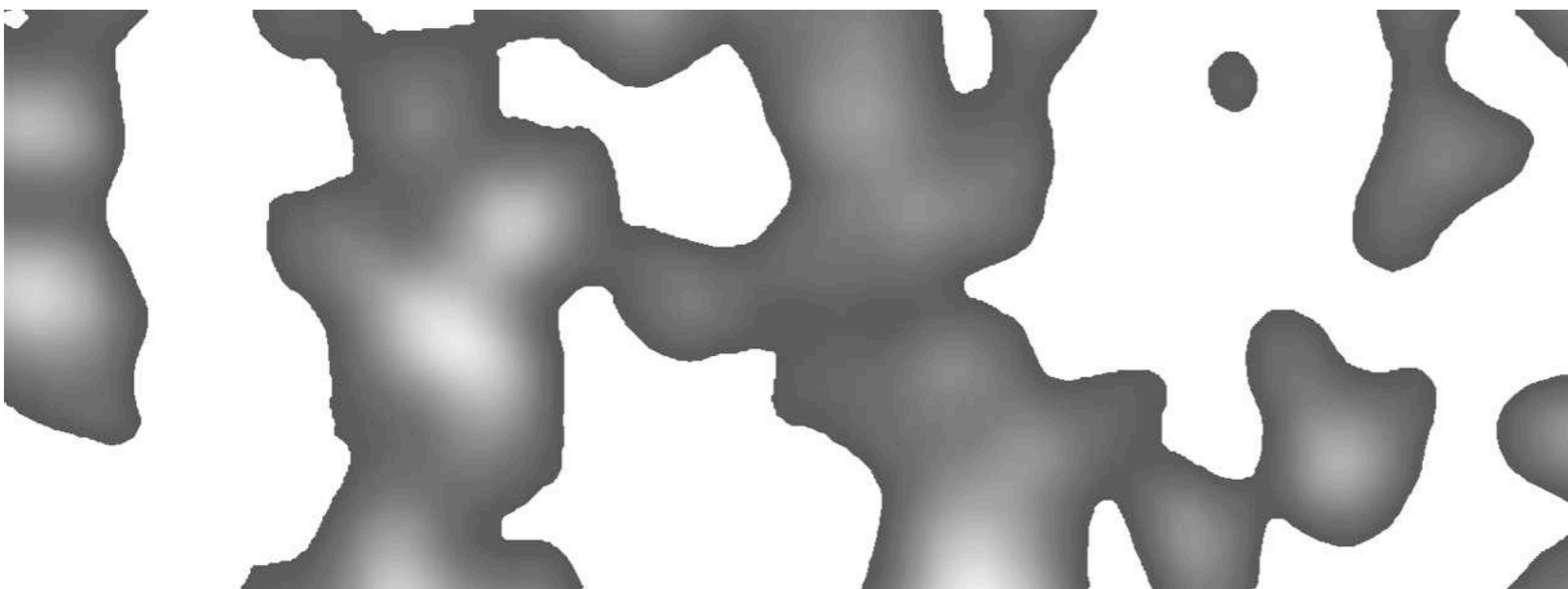
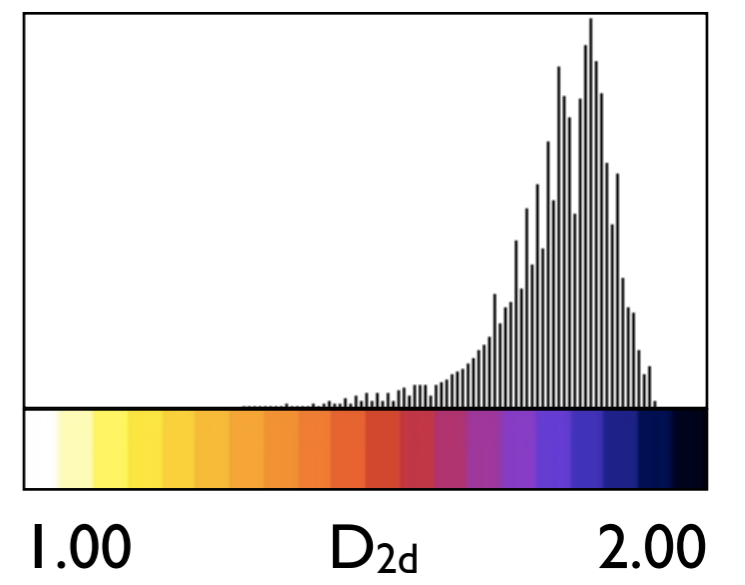
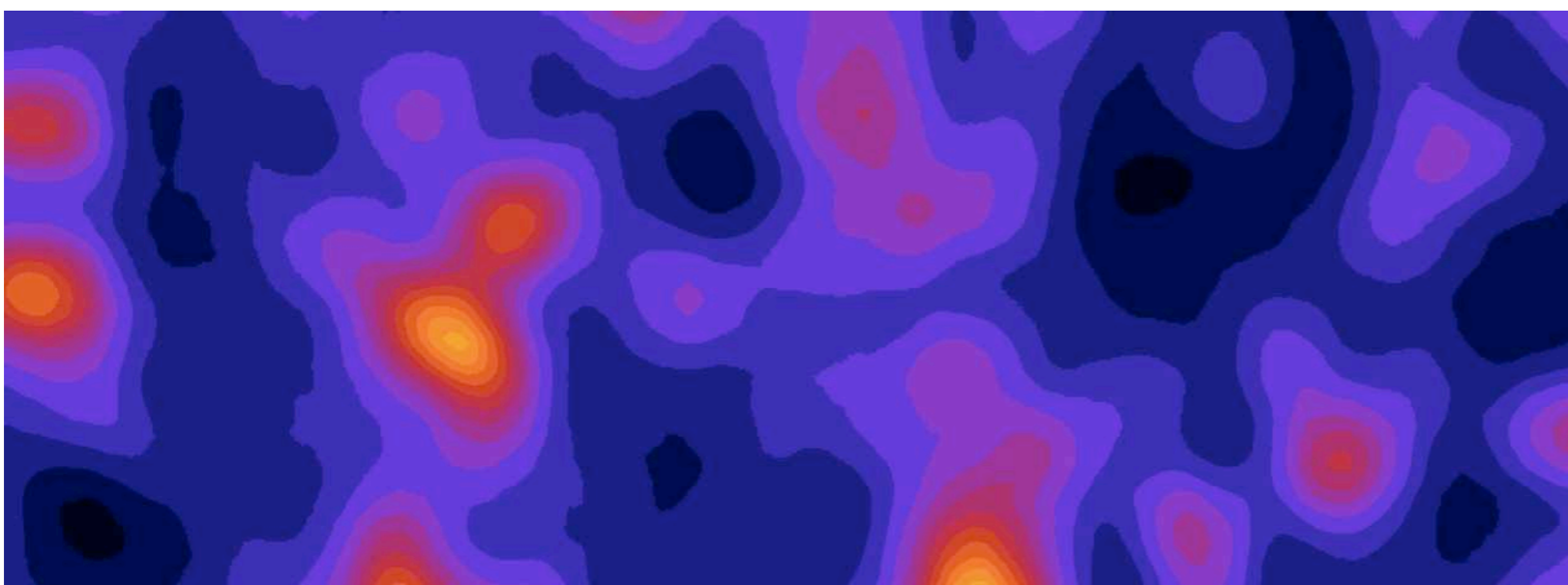
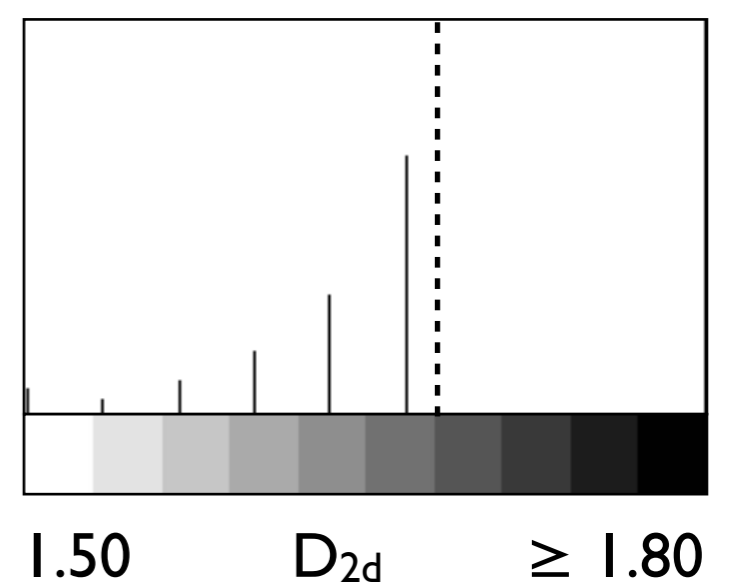
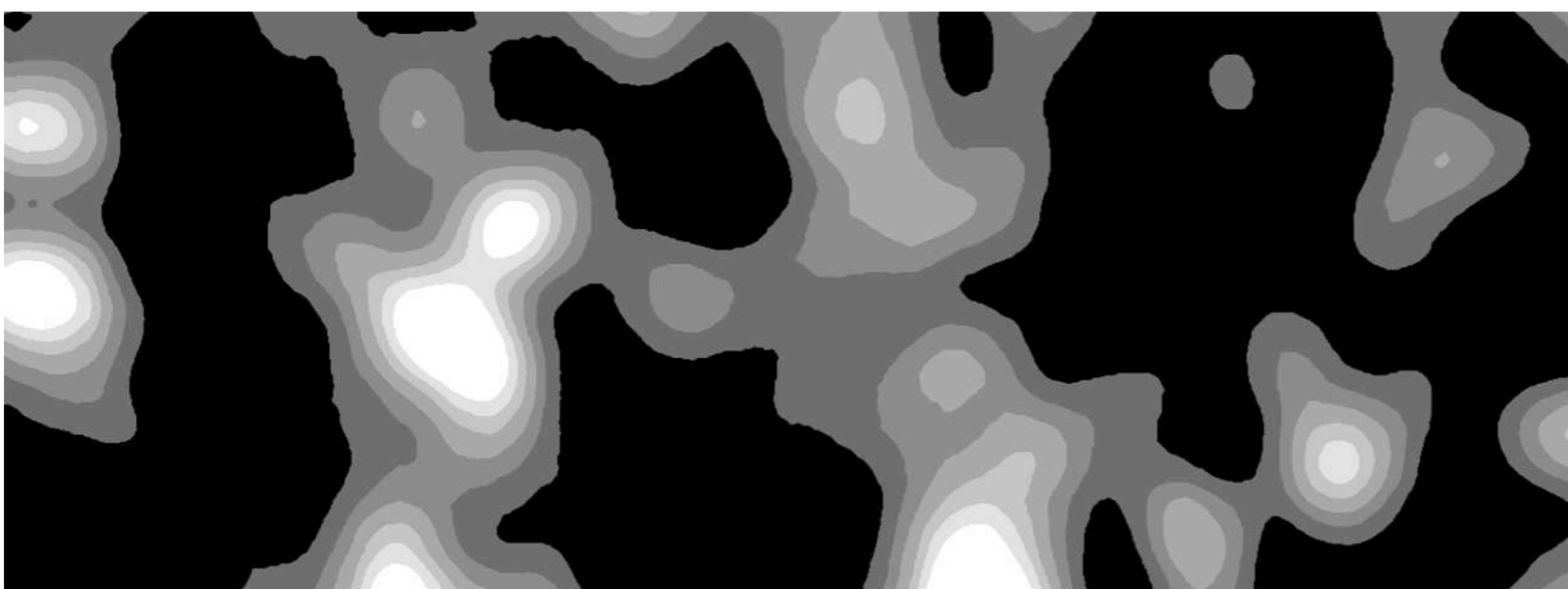
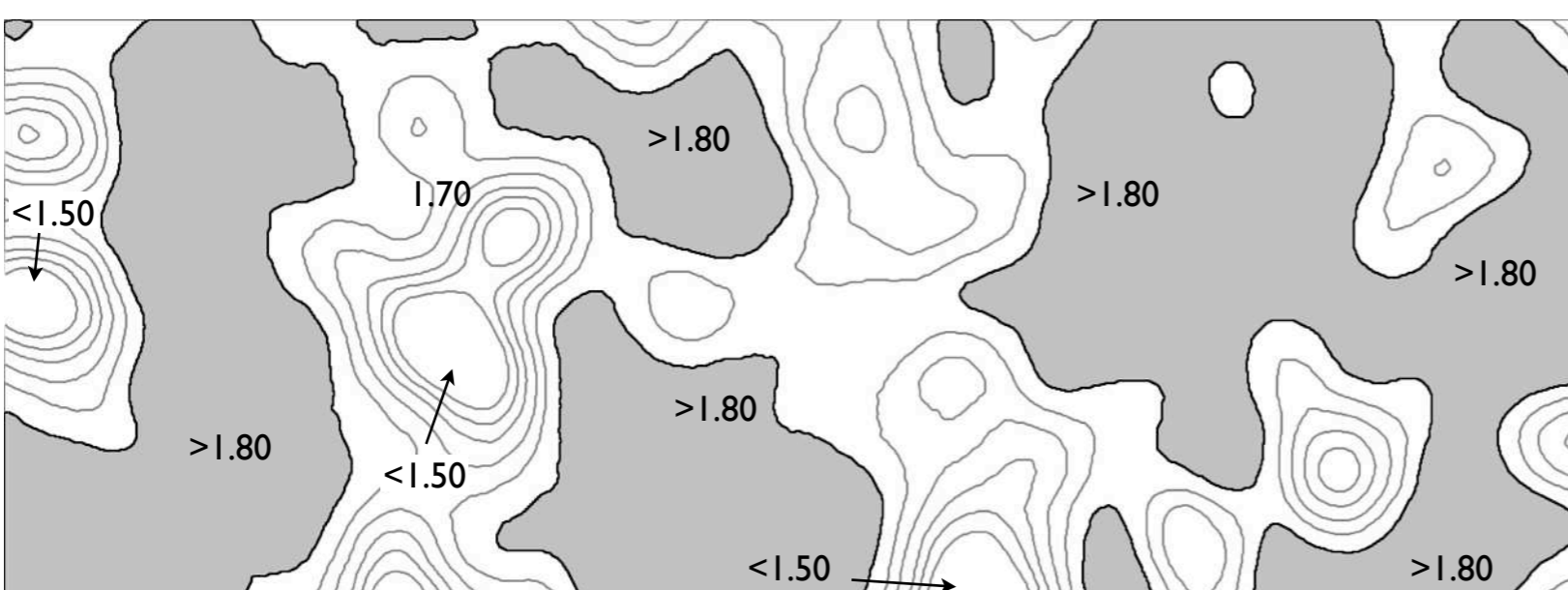
**Figure 13.20**

Visualizing matrix density maps.

(a) Scaled matrix density map, diameter of Gauss filter = 200 pixels;

(b) same as (a) seen with 20 gray levels, 50% and 65% contour are indicated in red and yellow, respectively;

(c) color coding of matrix content, lower cut-off = 10% (white), upper cut-off = 90% (black).

**a****b****c****d****e****f**

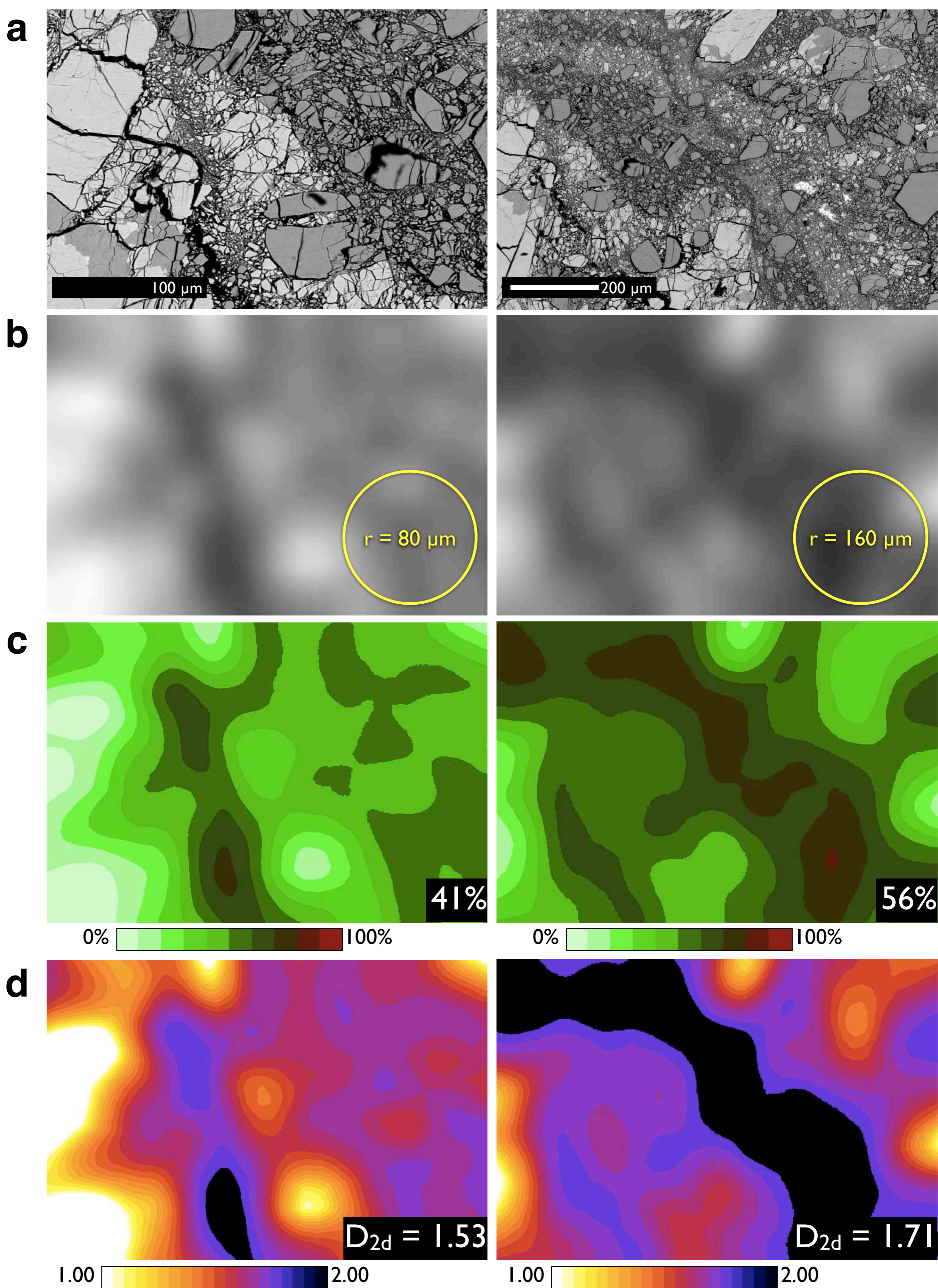


### **Figure 13.21**

Converting matrix density to fractal dimension.

- (a) Scaled map of fractal dimension (D map) obtained by applying LUT3 (for  $L_m = 1/8$ ) to matrix density image (Gauss filter  $d = 200$  pixels); average  $D_{2d} = 1.76$ ; LUT shown in upper left corner;
  - (b) same as (a) after stretching gray values to  $(1.00 < D_{2d} < 2.00)$ ;
  - (c) same as (a) after cropping to  $D_{2d} < 1.8$ ;
- for comparison, (a) and (b) shown with 20 gray levels;
- (d) same as (a) using 'Fire-2' LUT of Image SXM and setting LUT options to 20 colors;
  - (e) same as (c), after applying command [E] of the Lazy D map macro a second time ( $1.50 \leq D_{2d} \leq 2.00$ ), filling the white area with black, setting LUT options to 10 colors and applying LUT;
  - (f) contour map obtained from (e) using 'Find Edges' command (Process menu).

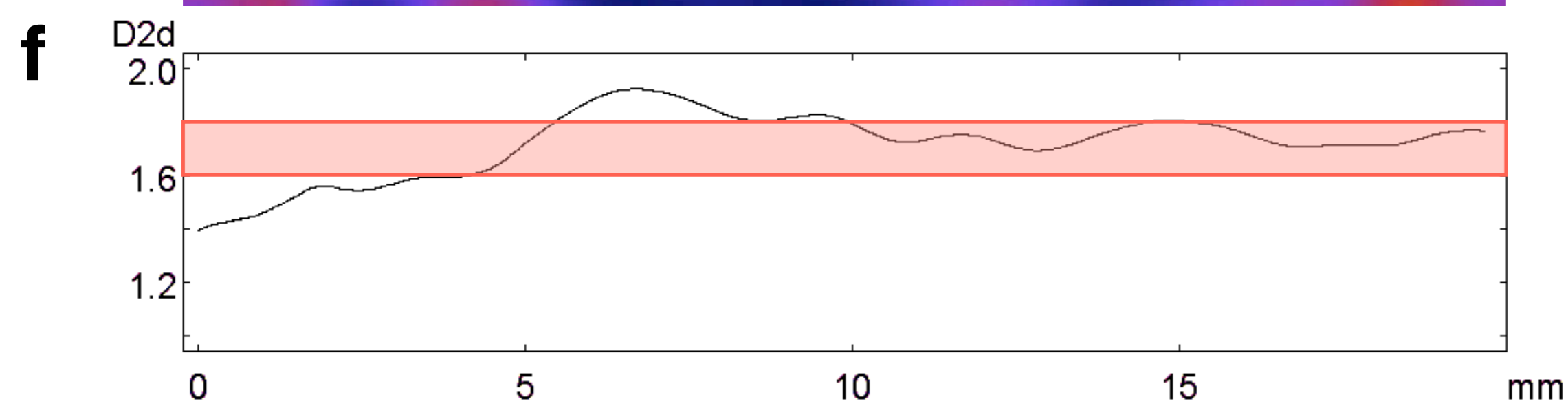
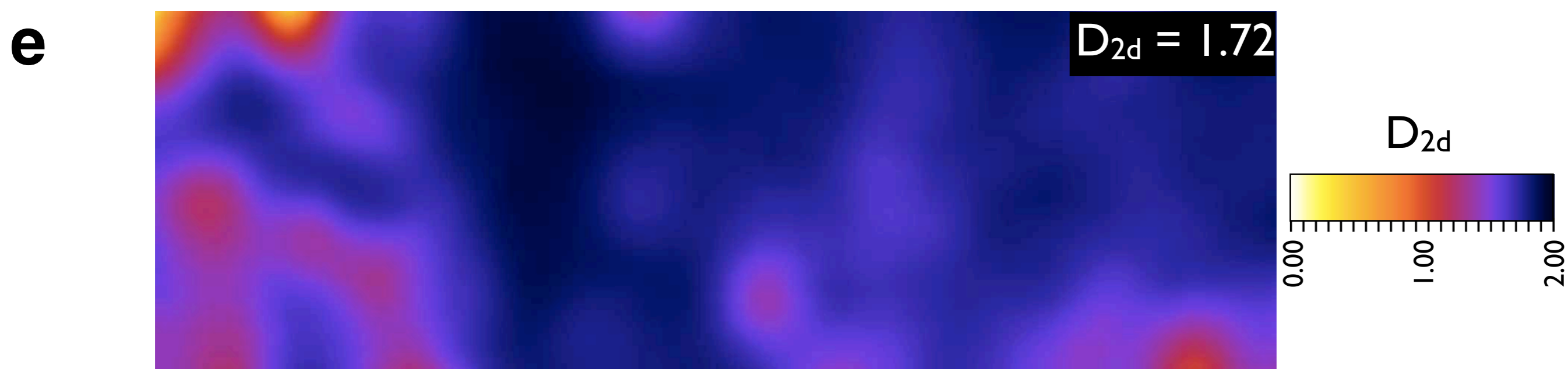
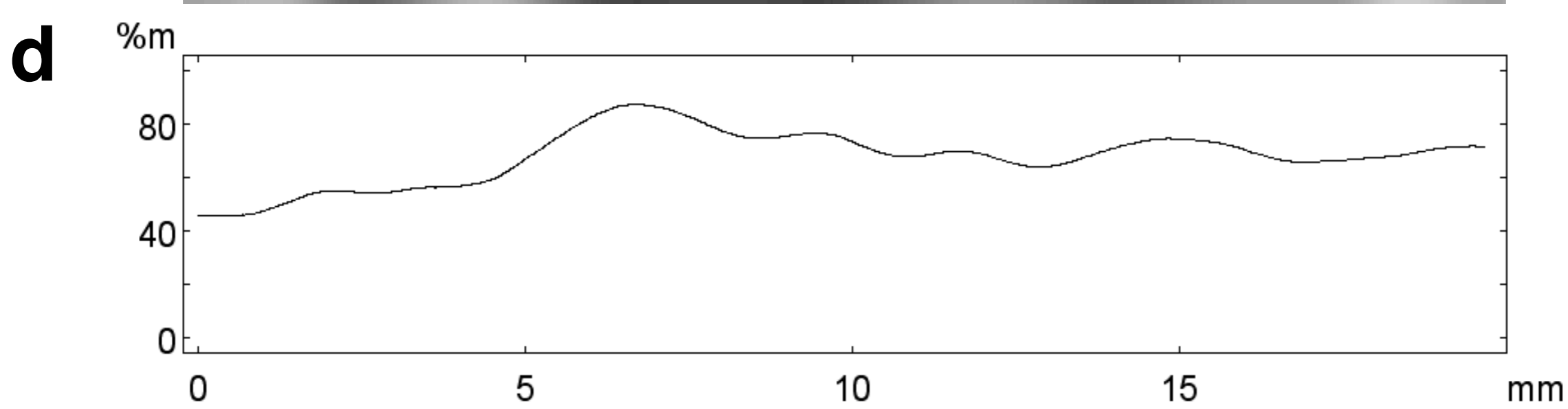
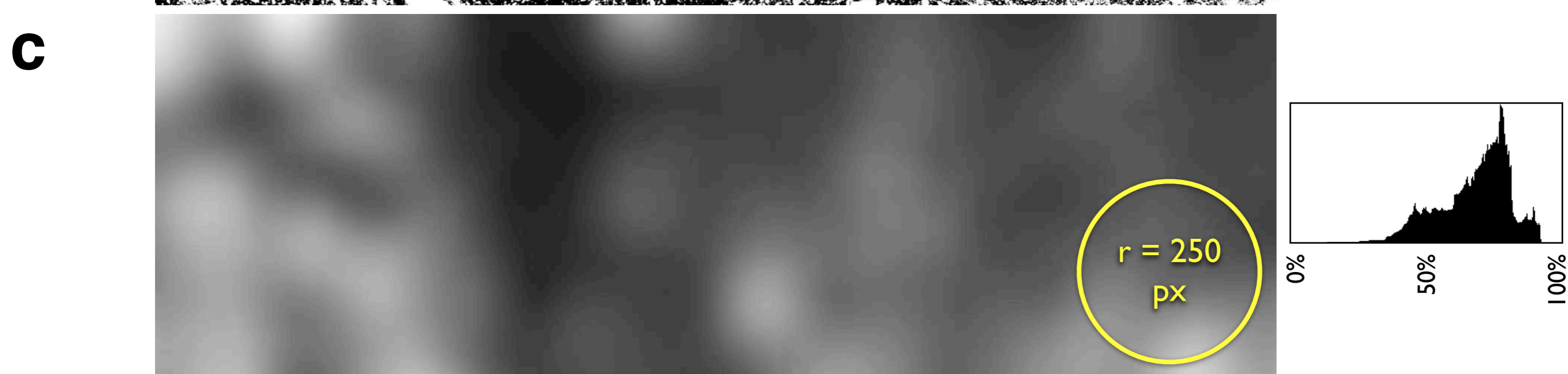
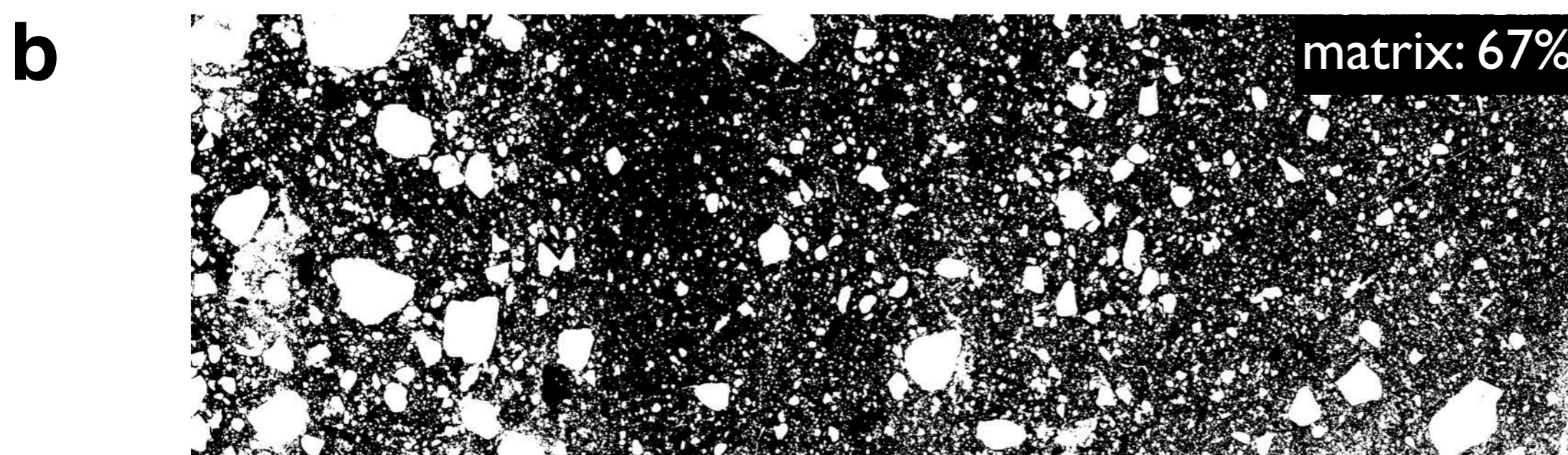
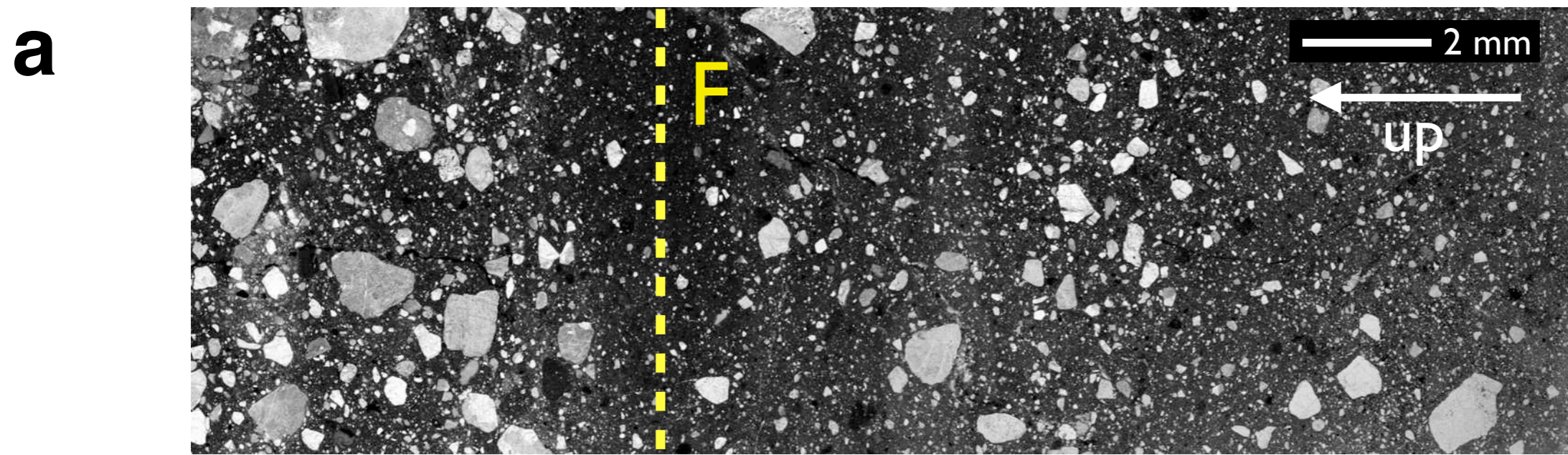




**Figure 13.22**

Two examples for D mapping. (a) SEM micrographs of experimentally produced fault at two different magnifications; (b) matrix density maps; size of Gauss filter is indicated by yellow circle; (c) color-coded matrix density maps; average matrix content is indicated; (d) D maps using LUT3 for a cut-off matrix grain size of  $L_m = 1/8$ ; average fractal dimension,  $D_{2d}$ , of grain size distribution is indicated.







**Figure 13.23**

D map of natural fault rock.

- (a) Thin section of natural fault rock taken from low angle detachment fault, F = trace of (horizontal) fault surface;
- (b) bitmap of (a); average density indicated;
- (c) matrix density map; histogram on right; size of Gauss filter indicated by yellow circle;
- (d) matrix density profile of (c), averaged over image height;
- (e) D map of (c) using LUT2 ( $L_m = 1/4$ ); color coding on right; average fractal dimension indicated;
- (f) D profile of (e), averaged over image height; range of values ( $1.60 \leq D_{2d} \leq 1.80$ ) is highlighted.