







Seeing two-dimensional objects in flatland. (a) With a central perspective; (b) under parallel projection.



Projections of a single ellipse.

The length of projection, P, depends on the orientation, α , of the ellipse with respect to the x-axis and the length of the short and long axes, b and a. Here, the axial ratio, b/a, is 0.5.



Projection curve, $P(\alpha)$, of a rotating ellipse.

The length of projection, P, depends on the initial orientation, α_i , of the ellipse and changes as a function of rotation, α_r ,

 $(0^{\circ} \le \alpha_r \le 180^{\circ})$. The shortest and the longest projections, $P(\alpha)_{min}$ and $P(\alpha)_{max}$, occur at angles α_{min} and α_{max} , respectively, when either the shortest or the longest axis, b or a, is parallel to the x-axis. Here, the axial ratio, b/a, is 0.5.



Projection curves of rotating ellipses with different axial ratios.

The ellipses are initially parallel to the x-axis, i.e., the initial orientation, α , = 0° and the long diameter, 2a, is 1.00. The projection curves, P(α), are shown for different axial ratios b/a of the ellipses: if b/a = 0, the ellipse is a line, if b/a = 1.00, the ellipse is a circle.





















a

d





С

f

Figure 14.5

Projection functions of elliptical and non-elliptical shapes.

Longest projections in red - shortest projections in green.

(a) Ellipse: one minimum, one maximum, minimum and maximum are 90° apart

(b) rhomb: two minima, one maximum

(c) rectangle: one minimum, two maxima

(d) octagon: four minima, four maxima

(e) oblique quadrangle: one minimum, one maximum, minimum and maximum are not 90° apart

(f) 'natural' shape: one minimum, one maximum, minimum and maximum are not 90° apart.



С

b

Angle

d

Figure 14.6

Long and short axes of elongate shapes.

(a) Original shapes with convex hull (stippled);

(b - d) best-fit ellipses with original outlines shown as dashed lines;

(b) the long axis, LA1, is defined as the longest projection, the short axis, SA2, as the projection normal to the longest;

(c) the short axis, SA1, is defined as the shortest projection, the long axis, LA2, as the projection normal to it;

(b, c) the angles, α_{i1} and α_{i2} , are the orientation of the long axes, LA₁ and LA₂, respectively, counterclockwise from positive x-axis;

(d) the axes and orientation of the best-fit ellipse are calculated using Image SXM Analyze.

	L - ellipse		S - ellipse		SXM - ellipse	
	a = LA_1 b = SA_2 α_{i1} = orientation of LA_1		b = SA ₁ a = LA ₂ α_{i2} = orientation of LA ₂		a = Majr b = Minr α = Angle	
shape	b/a	α _{il} (°)	b/a	α _{i2} (°)	b/a	α (°)
	0.50	0	0.50	0	0.50	0
2	0.80	153	0.50	0	0.50	0
3	0.40	162	0.33	0	0.38	167
4	0.42	28	0.40	42	0.38	35

Table 14.1

Best-fit ellipses.

For definition of LA₁, LA₂, SA₁, SA₂, α_{i1} , α_{i2} , see Figure 14.6; for definition of Majr, Minr and Angle, see Figure 8.2. Shapes are the same as in Figure 14.6; b/a = axial ratio; b = short axis; a = long axis; α_{i1} , α_{i2} , α = orientation of long axis, angles CCW from positive x-axis.



oolithic limestone















b

Outlines, convex hulls and best-fit ellipse approximations.

(a) Original fabric; top: particle in matrix = oolithic limestone (Hauptrogenstein, Jura Mountains Switzerland); bottom: crystalline aggregate = experimentally sheared Carrara marble (600°C, γ = 1.22, twinning regime, dextral shear sense); (b) convex hull of the particles;

(c) the axes and orientations of the best-fit ellipses are calculated using Image SXM Analyze.

(d) the long axes of the best-fit ellipses, LA₁, are defined as the longest, the short axes, SA₂, as the projections normal to the longest, the angles, α_{i1} are the orientations of the long axes, LA₁;

(e) the short axes of the best-fit ellipses, SA₁, are defined as the longest, the long axes, LA₂, as the projections normal to the shortest, the angles, α_{i2} are the orientations of the long axes, LA₂.

For better comparison with (c), the ellipses in (d) and (e) are also scaled to the size of the particles.



Projection curves, $B(\alpha)$, of sets of ellipses.

The projection curves of the individual ellipses (labeled 1 to 6) are shown as cumulative plots.

(a) Parallel ellipses: orientation distribution function (ODF) = delta function: $h(\alpha_i) = \infty$ if α_i = 30°; fabric as a whole has same anisotropy as individual ellipses

(b) ellipses with preferred orientation: ODF = normal distribution: $h(\alpha_i) = 30^\circ \pm 10^\circ$; fabric as a whole is less anisotropic than individual ellipses

(c) randomly oriented ellipses: ODF = uniform distribution; fabric as a whole is isotropic.



С





d

Figure 14.9

Line fabrics and projection curves $B(\alpha)$.

Orientation of lines is described in terms of normal distributions with mean orientation, $\mu = 0^{\circ}$ and different standard deviations.

(a) Perfectly parallel lines: $h(\alpha_i) = 1.00$; or normal distribution with $\mu = 0^\circ$; $\sigma = 0^\circ$;

- (b d) preferred orientation of lines: $h(\alpha_i)$ = normal distribution with $\mu = 0^\circ$; $\sigma = 10^\circ$, 20° and 30°;
- (e) random orientation of lines: $h(\alpha_i) = 1.00$; or normal distribution with $\mu = 0^\circ$; $\sigma = 90^\circ$;

(f) projection curve $B(\alpha)$ of sets of 100 lines; $B(\alpha)$ is normalized such that $B(\alpha)_{max} = 1.00$.





Effect of increasingly random orientation and increasing axial ratio on bulk fabric.

(a) Perfectly parallel lines: b/a = 0.00; $h(\alpha_i) = delta$ function;

(b) preferred orientation of lines: b/a = 0.00; $h(\alpha_i) = normal distribution$;

(c) random orientation of lines: b/a = 0.00; $h(\alpha_i) =$ uniform distribution;

(d) circles: b/a = 1.0;

(e) perfectly parallel ellipses: b/a = 0.5; $h(\alpha_i) = delta$ function;

(f) projection curves of fabrics (a - e); $B(\alpha)$ is normalized such that $B(\alpha)_{max} = 1.00$.

as1	72	COL	SC	m
95	12	cor		

gs172	x-y 1666	coordir 2	nates	smoothed
927. 928. 933. 936. 938. 939.	2470 2849 9293 1370 9568 9284	2 1 2 8 5 7	-5.596 -5.616 -6.116 -6.847 -8.097 -8.969	51385 06129 55819 79538 75637 09326
 926. 927. 9999	etc. 5576 2470 .000	8 1 0	-5.852 -5.596 9999	26130 51385 .0000
1009 1010 1013 1014 9999	etc .160 .903 .592 .400	8 9 4 3 0	-863.3 -862.6 -861.6 -861.3 9999	35156 58860 57352 39825 .0000



Software Box 14.1

Input file gs172.cor.scm with x-y coordinates of particle outlines; plot of file.

```
analysis of bulk particle fabric
maximum number of points per particles is 1000
maximum number of particles is 5000
 input file:
   line 1:
                      bti(132byte) title (must have)
   line 2:
                           total number of points
                      n
   for each particle: x,y
                             floating x-y coordinates
                                   ...etc.
                      Xend, Yend end coordinates
name of file >
gs172.cor.scm
end coordinate of input file (0, 9999, ... one number):
9999
increment of rotation angle (minimum: 1 deg.) >
10
do you want printout (0), file (1), both (2) ? >
name of output file ? [gs172.cor.p10] (return=default) >
name of file with B(alfa) curve ? [gs172.cor.j10] >
 name of file with long-axes ODF ? [gs172.cor.x10] >
name of file with particle data ? [gs172.cor.d10] >
```

Software Box 14.2

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8

Dialog with program PAROR; answers are numbered and highlighted, see text for explanation.

10°	5 °	۱°	file type
file.p10	file.p05	file.p01	screen output
file.d10	file.d05	file.d01	axes and orientations
file.j10	file.j05	file.j01	B(α) curve
file.x10	file.x05	file.x01	ODF of long axes

Table 14.2

Default file name extensions used for result files created by the PAROR program. For angular resolutions of 10° , 5° and 1° .

B(alfa)min Alfamin = Bulk b/a =	= 44.7 75.	62 0.94138	B(alfa)max = Alfamax =	47.549 14.
		0.91190		
Angular dif (diff < 90	ference = deg = dextral	119. monoclinic)		
Preferred o	orientation (of	LA1) alfap1 =	= 166.	
referred o	prientation (of	LA2) alfap2 =	= 15.	
Length of p	projections, B(alpha), (= Fer	et diam.)	
angle	mean	variance	st.dev.	skewness
10	47 51385	384 32730	19 60427	-0 23660
10 20	47.51385 47.45034	384.32730 376.00531	19.60427 19.39086	-0.23660 -0.24753
10 20 30	47.51385 47.45034 46.98285	384.32730 376.00531 361.55994	19.60427 19.39086 19.01473	-0.23660 -0.24753 -0.24991
10 20 30 40	47.51385 47.45034 46.98285 46.32403	384.32730 376.00531 361.55994 351.31671	19.60427 19.39086 19.01473 18.74344	-0.23660 -0.24753 -0.24991 -0.23435
10 20 30 40 50	47.51385 47.45034 46.98285 46.32403 45.60604	384.32730 376.00531 361.55994 351.31671 341.57437	19.60427 19.39086 19.01473 18.74344 18.48173	-0.23660 -0.24753 -0.24991 -0.23435 -0.21703
10 20 30 40 50 60	47.51385 47.45034 46.98285 46.32403 45.60604 45.08461	384.32730 376.00531 361.55994 351.31671 341.57437 335.89786	19.60427 19.39086 19.01473 18.74344 18.48173 18.32752	-0.23660 -0.24753 -0.24991 -0.23435 -0.21703 -0.20634
10 20 30 40 50 60 70	47.51385 47.45034 46.98285 46.32403 45.60604 45.08461 44.79977	384.32730 376.00531 361.55994 351.31671 341.57437 335.89786 335.37720	19.60427 19.39086 19.01473 18.74344 18.48173 18.32752 18.31331	-0.23660 -0.24753 -0.24991 -0.23435 -0.21703 -0.20634 -0.19058
10 20 30 40 50 60 70 80	47.51385 47.45034 46.98285 46.32403 45.60604 45.08461 44.79977 44.77297	384.32730 376.00531 361.55994 351.31671 341.57437 335.89786 335.37720 338.23099	19.60427 19.39086 19.01473 18.74344 18.48173 18.32752 18.31331 18.39106	-0.23660 -0.24753 -0.24991 -0.23435 -0.21703 -0.20634 -0.19058 -0.17372
10 20 30 40 50 60 70 80 90	47.51385 47.45034 46.98285 46.32403 45.60604 45.08461 44.79977 44.77297 45.08445	384.32730 376.00531 361.55994 351.31671 341.57437 335.89786 335.37720 338.23099 350.80429	19.60427 19.39086 19.01473 18.74344 18.48173 18.32752 18.31331 18.39106 18.72977	-0.23660 -0.24753 -0.24991 -0.23435 -0.21703 -0.20634 -0.19058 -0.17372 -0.19561
10 20 30 40 50 60 70 80 90 100	47.51385 47.45034 46.98285 46.32403 45.60604 45.08461 44.79977 44.77297 45.08445 46.10207	384.32730 376.00531 361.55994 351.31671 341.57437 335.89786 335.37720 338.23099 350.80429 362.16537	19.60427 19.39086 19.01473 18.74344 18.48173 18.32752 18.31331 18.39106 18.72977 19.03064	-0.23660 -0.24753 -0.24991 -0.23435 -0.21703 -0.20634 -0.19058 -0.17372 -0.19561 -0.19748
10 20 30 40 50 60 70 80 90 100 110	47.51385 47.45034 46.98285 46.32403 45.60604 45.08461 44.79977 44.77297 45.08445 46.10207 46.60590	384.32730 376.00531 361.55994 351.31671 341.57437 335.89786 335.37720 338.23099 350.80429 362.16537 367.72769	19.60427 19.39086 19.01473 18.74344 18.48173 18.32752 18.31331 18.39106 18.72977 19.03064 19.17623	-0.23660 -0.24753 -0.24991 -0.23435 -0.21703 -0.20634 -0.19058 -0.17372 -0.19561 -0.19748 -0.19149
10 20 30 40 50 60 70 80 90 100 110 120	47.51385 47.45034 46.98285 46.32403 45.60604 45.08461 44.79977 44.77297 45.08445 46.10207 46.60590 46.86028	384.32730 376.00531 361.55994 351.31671 341.57437 335.89786 335.37720 338.23099 350.80429 362.16537 367.72769 366.88049	19.60427 19.39086 19.01473 18.74344 18.48173 18.32752 18.31331 18.39106 18.72977 19.03064 19.17623 19.15413	-0.23660 -0.24753 -0.24991 -0.23435 -0.21703 -0.20634 -0.19058 -0.17372 -0.19561 -0.19748 -0.19149 -0.18453
10 20 30 40 50 60 70 80 90 100 110 120 130	47.51385 47.45034 46.98285 46.32403 45.60604 45.08461 44.79977 44.77297 45.08445 46.10207 46.60590 46.86028 47.05245	384.32730 376.00531 361.55994 351.31671 341.57437 335.89786 335.37720 338.23099 350.80429 362.16537 367.72769 366.88049 369.35434	19.60427 19.39086 19.01473 18.74344 18.48173 18.32752 18.31331 18.39106 18.72977 19.03064 19.17623 19.15413 19.21859	-0.23660 -0.24753 -0.24991 -0.23435 -0.21703 -0.20634 -0.19058 -0.17372 -0.19561 -0.19748 -0.19149 -0.18453 -0.18661
10 20 30 40 50 60 70 80 90 100 100 110 120 130 140	47.51385 47.45034 46.98285 46.32403 45.60604 45.08461 44.79977 44.77297 45.08445 46.10207 46.60590 46.86028 47.05245 47.23512	384.32730 376.00531 361.55994 351.31671 341.57437 335.89786 335.37720 338.23099 350.80429 362.16537 367.72769 366.88049 369.35434 372.40295	19.60427 19.39086 19.01473 18.74344 18.48173 18.32752 18.31331 18.39106 18.72977 19.03064 19.17623 19.15413 19.21859 19.29774	-0.23660 -0.24753 -0.24991 -0.23435 -0.21703 -0.20634 -0.19058 -0.17372 -0.19561 -0.19748 -0.19149 -0.18453 -0.18661 -0.18645
10 20 30 40 50 60 70 80 90 100 100 110 120 130 140 150	47.51385 47.45034 46.98285 46.32403 45.60604 45.08461 44.79977 44.77297 45.08445 46.10207 46.60590 46.86028 47.05245 47.23512 47.32745	384.32730 376.00531 361.55994 351.31671 341.57437 335.89786 335.37720 338.23099 350.80429 362.16537 367.72769 366.88049 369.35434 372.40295 376.34705	19.60427 19.39086 19.01473 18.74344 18.48173 18.32752 18.31331 18.39106 18.72977 19.03064 19.17623 19.15413 19.21859 19.29774 19.39967	-0.23660 -0.24753 -0.24991 -0.23435 -0.21703 -0.20634 -0.19058 -0.17372 -0.19561 -0.19748 -0.19149 -0.18453 -0.18661 -0.18645 -0.17898
10 20 30 40 50 60 70 80 90 100 100 110 120 130 140 150 160	47.51385 47.45034 46.98285 46.32403 45.60604 45.08461 44.79977 44.77297 45.08445 46.10207 46.60590 46.86028 47.05245 47.05245 47.23512 47.32745 47.15906	384.32730 376.00531 361.55994 351.31671 341.57437 335.89786 335.37720 338.23099 350.80429 362.16537 367.72769 366.88049 369.35434 372.40295 376.34705 379.88049	19.60427 19.39086 19.01473 18.74344 18.48173 18.32752 18.31331 18.39106 18.72977 19.03064 19.17623 19.15413 19.21859 19.29774 19.39967 19.49052	-0.23660 -0.24753 -0.24991 -0.23435 -0.21703 -0.20634 -0.19058 -0.17372 -0.19561 -0.19748 -0.19149 -0.18453 -0.18661 -0.18645 -0.17898 -0.16303
10 20 30 40 50 60 70 80 90 100 100 110 120 130 140 150 160 170	47.51385 47.45034 46.98285 46.32403 45.60604 45.08461 44.79977 44.77297 45.08445 46.10207 46.60590 46.86028 47.05245 47.05245 47.23512 47.32745 47.15906 46.72064	384.32730 376.00531 361.55994 351.31671 341.57437 335.89786 335.37720 338.23099 350.80429 362.16537 367.72769 366.88049 369.35434 372.40295 376.34705 379.88049 378.98520	19.60427 19.39086 19.01473 18.74344 18.48173 18.32752 18.31331 18.39106 18.72977 19.03064 19.17623 19.15413 19.21859 19.29774 19.39967 19.49052 19.46754	-0.23660 -0.24753 -0.24991 -0.23435 -0.21703 -0.20634 -0.19058 -0.17372 -0.19561 -0.19748 -0.19149 -0.18453 -0.18661 -0.18645 -0.17898 -0.16303 -0.15489

histogram: average length of projection versus angle of projection

2

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J
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10		••••***	*************************
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170			<*************************************
180		••••**	<*************************************

Software Box 14.3

PAROR output file gs172.cor.p10:

(1) Calculation of $B(\alpha)_{max}$, $B(\alpha)_{min}$, α_{max} and α_{min} using an evaluation at 1° increments, angular difference is between α_{max} and α_{min} ;

(2) Length of projection, $B(\alpha)$, list of values;

(3) Mean length of projection and standard deviation, shown as histogram;

number of projected particles: 343 mean variance st.dev. skewness longest projection 51.8276 430.4483 20.7472 -0.1907 shortest projection 39.6860 279.0329 16.7043 -0.2845 perp. to longest 41.4572 298.4258 17.2750 -0.3207 perp. to shortest 50.6558 426.2610 20.6461 -0.1372 perp.L/longest 0.8053 0.0207 0.1440 -1.1211 shortest/longest 0.7680 0.0183 0.1352 -1.0944 shortest/perp.S 0.7905 0.0212 0.1456 -1.0217 perp.L/perp.S 0.7905 0.0212 0.1456 -1.0217 perp.L/perp.S 0.8294 0.0249 0.1578 -0.9547 histogram of long axes and number of long axes: number of long axes: 25.272 30 ************************************	evaluation of particle of	ixes:			
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52.72 30 *********************************	43.88 20 ************	* * * * * * * * * * * * * * * * * * *	16 20 *****	• • • • • • • • • • • • • • • • • • •	
52.30 40 *******************************	52.72 30 ************	,	26 30 ****	• • • • • • • • • • • • • • • • • • •	
$\begin{array}{cccccccccccccccccccccccccccccccccccc$	52.30 40 ************	,	23 40 *****	• • • • • • • • • • • • • • • • • • •	
43.69 60 ************************************	58.02 50 ************	, , , , , , , , , , , , , , , , , , ,	18 50 *****	, , , , , , , , , , , , , , , , , , ,	
53.99 70 ************************************	43.69 60 ************	~~~~~~~~~	20 60 ****	,	
57.62 80 ************************************	53.99 /0 ***********************************	*****		*****	
43.48100*********************************	CO 42 OO ***************	****	LO 80 *****	****	
45.48100*********************************	00.45 90 ***********************************	****	13 90 ****	****	
39.40 110 110 110 111 120 111 120 111 120 111 120 111 120 111 120 111 120 <td< td=""><td>45.40 100 *********************************</td><td>****</td><td>1/ 110 ****</td><td>****</td><td></td></td<>	45.40 100 *********************************	****	1/ 110 ****	****	
55.32 130 ************************************	<i>A</i> 1 06 120 ***********************************	****	11 170 ****	****	
53.32 130 130 140 130 140 130 140 <td< td=""><td>55 37 130 ************************************</td><td>****</td><td>14 120 ****</td><td>*****</td><td></td></td<>	55 37 130 *** *********************************	****	14 120 ****	*****	
48.31 150 ************************************	54 37 140 ***********************************	*****	24 140 ****	****	
51.23 160 ************************************	48 31 150 **********************************	****	21 150 ****	****	
58.36 170 ************************************	51 23 160 *************	*****	41 160 ****	*****	****
56.10 180 *********************************	58.36 170 ************	*****	32 170 ****	*****	
	56.10 180 *************	*****	8 180 ****	k	

histogram of long axes and number of long axes LA2 versus angle of rotation

6			average length of long axes:			number of long axes:
	54.29	10	******	33	10	*****
	49.71	20	*****	28	20	******
	59.64	30	*******	27	30	*******
	53.75	40	******	27	40	*******
	56.98	50	******	16	50	*****
	62.63	60	*****	8	60	****
	58.97	70	*****	8	70	****
	51.08	80	*****	7	80	****
	34.83	90	*****	31	90	*****
	50.66	100	*****	28	100	*****
	50.02	110	*****	16	110	*****
	43.84	120	*****	10	120	*****
	62.69	130	*******	11	130	*****
	58.40	140	******	10	140	*****
	56.56	150	*******	13	150	*****
	47.53	160	*****	15	160	*****
	56.79	170	******	13	170	*****
	40.26	180	*****	42	180	******

Software Box 14.3

4

5

PAROR output file gs172.cor.p10:

- (4) Axes and axial ratios of particles;
- (5) Length and number of long axes LA_1 versus orientation;
- (6) Length and number of long axes LA₂ versus orientation.

angle	rel_length_of_proj
0	0.98680
10	1.00000
20	0.99866
30	0.98882
40	0.97496
50	0.95985
60	0.94887
70	0.94288
80	0.94231
90	0.94887
100	0.97029
110	0.98089
120	0.98624
130	0.99029
140	0.99413
150	0.99608
160	0.99253
170	0.98331
180	0.98680



Software Box 14.4 PAROR output file gs172.cor.j10:Values of $B(\alpha)$ and plot.

angle	rel_leng	jth_LA1	rel_length_LA2
-1	80.0	0.21367	0.94366
-1	75.0	0.00000	0.00000
-1	70.0	0.15845	1.00000
-1	65.0	0.00000	0.00000
-1	60.0	0.33429	0.77693
-1	55.0	0.00000	0.00000
-1	50.0	0.65264	0.89880
-1	45.0	0.00000	0.00000
-1-	40.0	0.57269	0.80994
etc	•		
1	35.0	0.00000	0.00000
1	40.0	0.62068	0.32597
1	45.0	0.00000	0.00000
1	50.0	0.48298	0.41040
1	55.0	0.00000	0.00000
1	60.0	1.00000	0.39790
1	65.0	0.00000	0.00000
1	70.0	0.88910	0.41206
1	75.0	0.00000	0.00000
1	80.0	0.21367	0.94366



Software Box 14.5

PAROR output file gs172.cor.x10: Length weighted orientation distributions (ODF) of long axes, LA₁ and LA₂, and rose diagrams. Intermediate zero values (at 5°, 15°, etc.) are inserted to facilitate plotting of the rose diagrams.

paror analysis of gs172.cor.scm

number of particles: 343

minimum number of points/particle : 3
maximum number of points/particle : 1000
increment of rotation : 10

13.7793

22.9016

31.3816

40.1192

16.5276

65.3091

22.2847

5.3902

25.5773

35.0314

42.0580

60.0673

16.7043

67.8792

28.2261

5.7143

13.6912

21.2694

29.2222

38.3636

4.7959

14.7920

64.0367

20.4206

longest(LA1) shortest(SA1) perp.L(SA2) perp.S(LA2) SA2/LA1 SA1/LA1 alfaLA1 alfaLA2 Хс mkl Yc # 29.5026 24.8078 29.2895 0.9928 0.8409 930.24 -18.73 1 28.2858 140. 100. 40. 2 3.8694 3.7931 0.8348 0.8156 -7.01 4.6351 3.7804 40. 0. 40. 1254.75 29.8744 27.5646 28.0435 0.8994 1269.88 -20.06 3 31.1801 0.8840 150. 0. -30. 4 29.9597 24.5553 25.2841 29.8036 0.8439 0.8196 60. 50. 1202.01 -20.69 10. 5 18.5911 13.4676 13.7404 18.2917 0.7391 0.7244 120. 580.51 -20.60 110. -10. 6 63.9011 60.3972 62.8714 0.9452 -43.85 59.7227 0.9346 100. 40. 60. 555.33 44.6852 7 46.6194 38.0730 38.2518 0.8205 0.8167 130. 110. 792.63 -36.75 20. 8 20.7269 21.7226 23.7549 1145.09 -29.18 0.8914 0. -10. 24.3700 0.8505 170. 9 8.9011 8.9036 0.6763 0.6761 -26.89 13.1649 13.0755 50. 40. 10. 340.97 10 72.8184 66.3386 68.1621 69.0571 0.9361 694.44 -56.23 70. 0.9110 30. 40. 0.7031 8.5161 5.7747 5.9877 7.7660 -25.83 11 0.6781 160. 0. -20. 1164.60 12 63.8346 43.2561 46.0511 63.4958 0.7214 0.6776 -45.41 0. 10. -10. 1236.91 16.1638 14.7670 0.9189 13.7806 14.8526 0.8526 1072.67 -31.97 13 60. 90. -30. 22.4911 14.0573 22.3257 0.6250 -35.82 12.9760 14 0.5769 110. 120. -10. 729.91 34.8403 22.8076 22.8076 34.8403 0.6546 0.6546 187.99 -45.67 15 100. 100. 0. 56.1935 16 52.2959 54.6604 0.9306 0.8921 50.1285 70. 50. 20. 275.06 -53.77 44.3838 34.9235 35.6008 44.0069 0.8021 0.7869 -48.03 17 80. 90. -10. 434.05

0.5359

0.6387

0.7305

0.6657

0.8938

0.9571

0.9345

0.7472

0.5325

0.5932

0.6803

0.6366

0.7953

0.8566

0.9163

0.6847

90.

30.

110.

160.

140.

110.

60.

70.

100.

100.

10.

150.

90.

90.

40.

100.

-10.

10.

20.

10.

50.

20.

20.

-30.

486.00

511.97

1303.04

614.09

1172.68

1091.26

331.27

467.30

-40.85

-46.75

-45.06

-53.61

-33.68

-41.03

-71.11

-53.31

.... etc.

18

19

20

21

22

23

24

25

25.7133

35.8567

42.9562

60.2673

6.0305

17.2685

69.8852

29.8251

Software Box 14.6

PAROR output file gs172.cor.d10:Axes, axial ratios and orientations of particles:

= number of analyzed particle;

longest (LAI) = longest projection, shortest (SAI) = shortest projection of particle;

perp.(SA2) = projection perpendicular to longest, perp.(LA2) = projection perpendicular to shortest projection of particle;

SA2/LAI = axial ratio (= short / long axis) based on LAI = longest projection;

SAI/LAI = minimum possible value for axial ratio;

alfaLAI = orientation of longest projection;

alfaLA2 = orientation of normal to shortest projection;

mkl = angular difference between the orientations of LA_1 and LA_2 ;

 X_c and Y_c = center point coordinates of particles calculated as the average x- and the average y-coordinate of the points digitized on the outline.

Angles are CCLW from positive x-axis.



Natural examples for particle fabric analysis.

(a) Oolithic limestone of the Hauptrogenstein formation, Jura Mountains, polished surface;

(b) experimentally deformed Carrara marble, thin section, polarized light, sample CTI 600°, intracrystalline slip regime, γ

= 1.22.









d

b



Figure 14.12

PAROR analysis of an oolithic limestone.

(a) Bitmap of fabric (prepared from acetate peel of GS 172);

(b) projection curve $B(\alpha)$ of gs172.cor.scm;

(c) rose diagrams of long axes of particles (5° resolution) using LA₁ axes(longest projections of particles);

(d) same as (c) using LA_2 axes (= projections normal to the shortest projection).

The bulk preferred orientations, α_{p1} (green) and α_{p2} (red), calculated from α_{max} and α_{min} , respectively, are superposed on the rose diagrams. (The exact values for b/a, α_{p1} and α_{p2} were derived from analyses made with 1° increment of rotation).









d

b



Figure 14.13

PAROR analysis of an experimentally sheared marble.

(a) Bitmap of fabric (prepared from sample CTI);

(b) projection curve $B(\alpha)$ of CTL.cor.scm;

(c) rose diagrams of long axes of particles (5° resolution) using LA₁ axes (longest projections of particles);

(d) same as (c) using LA_2 axes (= projections normal to the shortest projection).

The bulk preferred orientations, α_{p1} (green) and α_{p2} (red), calculated from α_{max} and α_{min} , respectively, are superposed on the rose diagrams. (The exact values for b/a, α_{p1} and α_{p2} were derived from analyses made with 1° increment of rotation).

a



b

Figure 14.14

Over-packed oolithic limestone with grain-to-grain pressure solution contacts.

(a) Analyzed area of thin section, inverted contrast, showing packing of ooides.

(b) entire thin section (area shown = $21 \cdot 32 \text{ mm}^2$), transmitted light; rectangle denotes analyzed area. (Sample courtesy Samuel Mock).

С

Figure 14.15

Ooides and pressure solution grain-to-grain contacts.

(a) Bitmap of ooides (b) bitmap of contact surfaces;

(c) outlines of ooides (d) outlines of contact surfaces.

d

270

Figure 14.16

С

PAROR analysis of ooides and grain-to-grain contacts.

(a) projection curve $B(\alpha)$ of the ooides (outlines in file oo.cor.scm);

(b) projection curve $B(\alpha)$ of contact areas (outlines in ps.cor.scm);

- (c) rose diagrams of long axes of ooides (results file oo.cor.r05, 5° resolution):
- (d) rose diagrams of long axes of contact areas (result file ps.cor.r05, 5° resolution): $LA_2 = long$ axes defined by the projection normal to the shortest projection.

Superposed in (c) to (d) are the preferred orientations, α_{p1} (red) and α_{p2} (green), derived from α_{max} and α_{min} ; the exact values for b/a, α_{min} and α_{max} were derived from analyses made with 1° increment of rotation.

Restoring outlines - recreating 'fossil' fabrics.

(a) From left to right: Bitmap of present day ooides with pressure solution grain-to-grain contact; outlines of restored ooides; outlines of pressure solution contacts;

(b) to (d) PAROR analysis of present day ooides (b), of restored ooides (c) and of pressure solution contacts (d); particle projection, $B(\alpha)$, is shown on left; ODF of long axes, LA₁, on right; preferred orientations, α_{p1} (green) and α_{p2} , (red) are superposed on rose diagram, the value for $b/a = B(\alpha)_{min} / B(\alpha)_{max}$, is indicated.

Concept of bounding box.

At any given orientation of the outline, the width of the bounding box, $bbw(\alpha)$, corresponds to the length of projection

on the x-axis, i.e., to $P(\alpha)$; the height of the bounding box, $bbh(\alpha)$, to the projection on the y-axis, i.e., to $P(\alpha + 90^{\circ})$.

Comparison of PAROR and bounding box results for sample CTI (Figure 14.13).

Left: projection curves $B(\alpha)$: top: calculated using bounding boxes of Image SXM at 10° intervals; bottom: using PAROR projections at 1° interval (bottom);

Right: rose diagrams of long axes of particles (5° resolution): top: ODF of the long axes of SXM best-fit ellipses; bottom: ODF of the longest projections LA₁ and projections normal to the shortest LA₂.

All rose diagrams are length weighted; in blue: average orientation of the long axes of the SXM ellipses; in green and red: the bulk preferred orientations, α_{p1} and α_{p2} , calculated from α_{max} and α_{min} , respectively, and superposed on the rose diagrams; the exact values for b/a, α_{p1} and α_{p2} were derived from analyses made with 1° increment of rotation.