Figure 19.1
One dimensional spatial frequencies.
Superposition of different sine waves; image (left) and gray value profile (right) are shown for:
(a) fundamental frequency: sin(x); (b) sin(x) plus 3 harmonics: sin(x) + sin(3x)/3 + sin(5x)/5 + sin(7x)/7; (c) sin(x) plus 7 harmonics: sin(x) + sin(3x)/3 + ... + sin(15x)/15; (d) rectangular wave as limiting case for infinite number of harmonics: sin (x) + sin(3x)/3 + ... ; (e) periodic wave pattern - 8 repetitions, (f) general wave pattern - no periodicity within width of image.

Figure 19.2
Two dimensional spatial frequencies.
Images as superposition of wave trains; image (left) and gray value profile (right).
(a) Superposition of horizontal and vertical sine wave; horizontal gray value profile; (b) vertical gray value profile of (a); (c) superposition of a number of sine waves: horizontal gray value profile; (d) oblique gray value profile of (c); (e) image of fern leaves: horizontal gray value profile; (f) vertical gray value profile of (e).

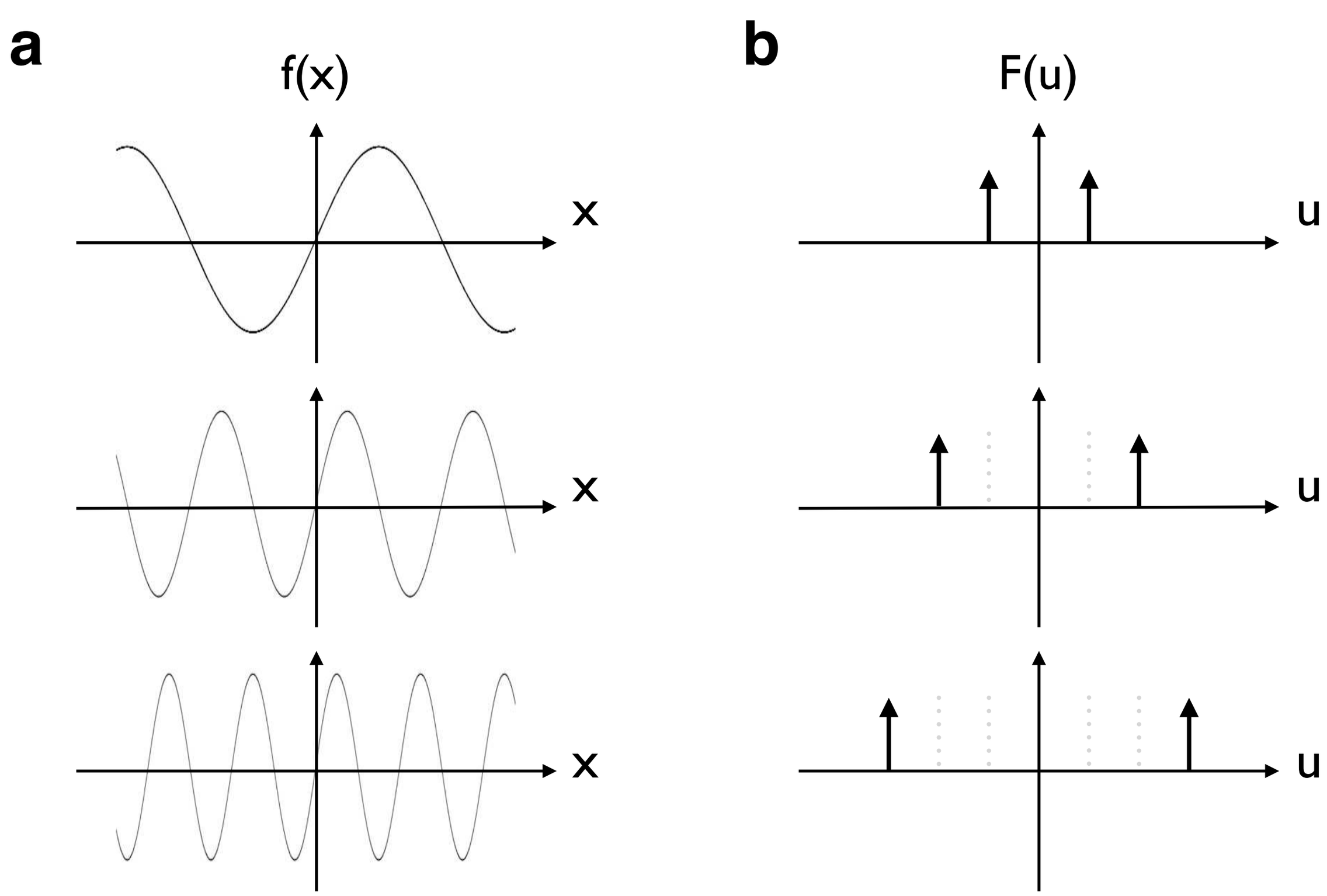
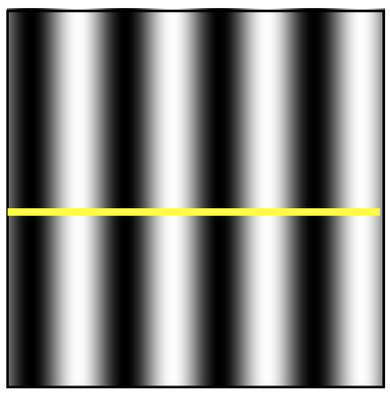
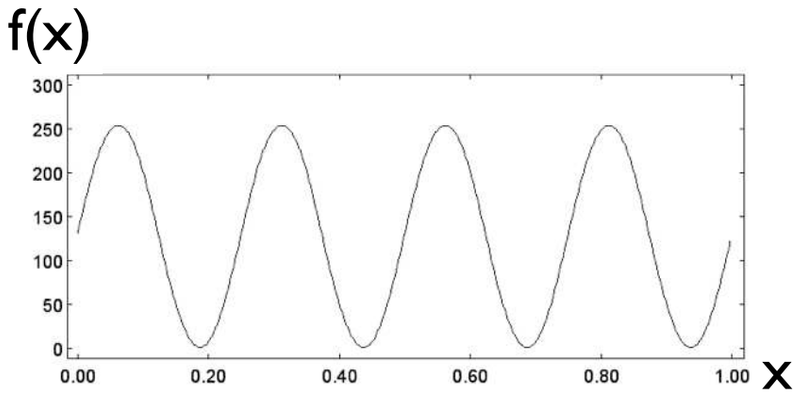
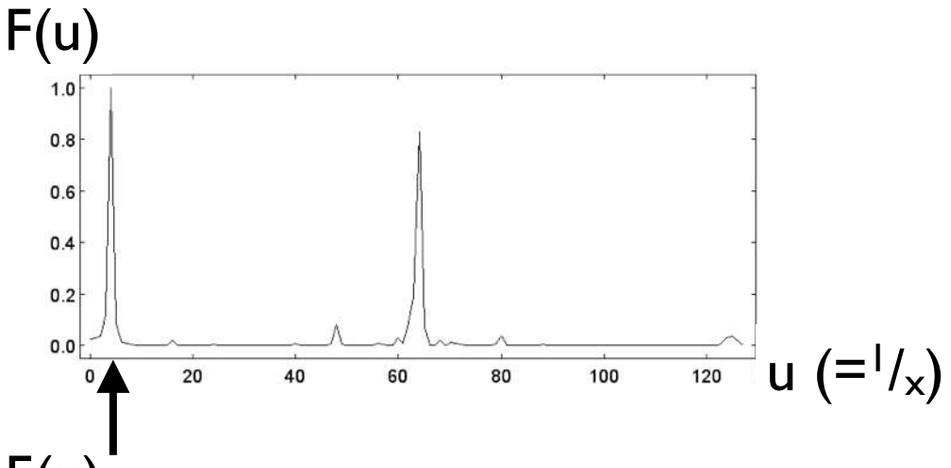


Figure 19.3
Fourier transform of sine waves.
(a) Sine waves with decreasing wavelength, i.e., increasing frequency (top to bottom);
(b) corresponding Fourier transforms.
x = length; u = frequency = 1/x; F(u) = Fourier transform of f(x).

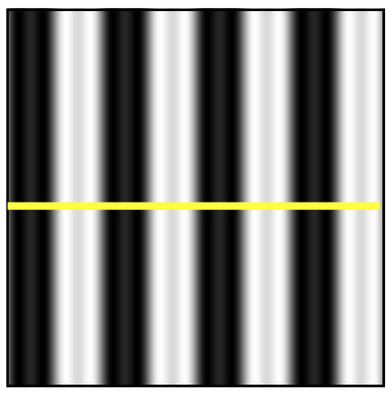
a

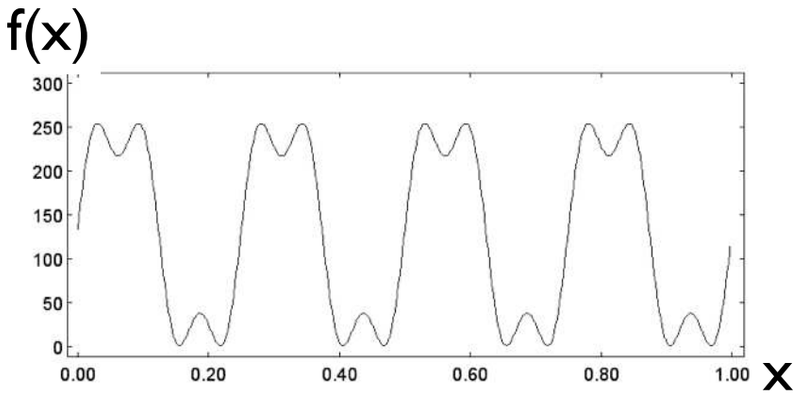


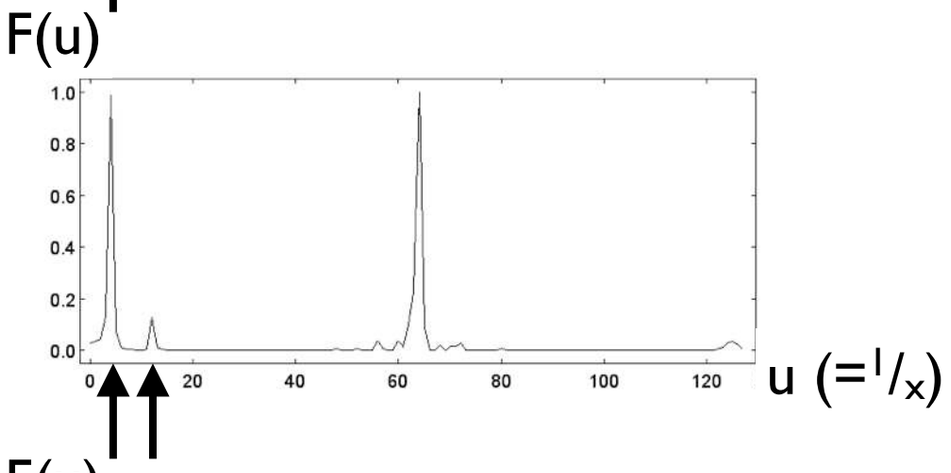

f(x)
x


F(u)
u (=1/x)

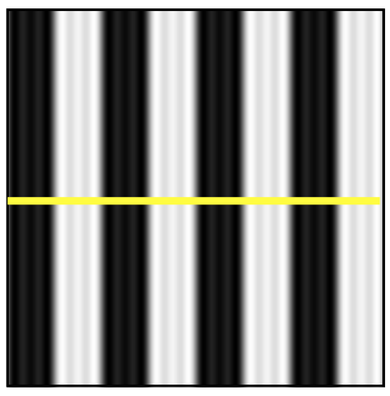
b

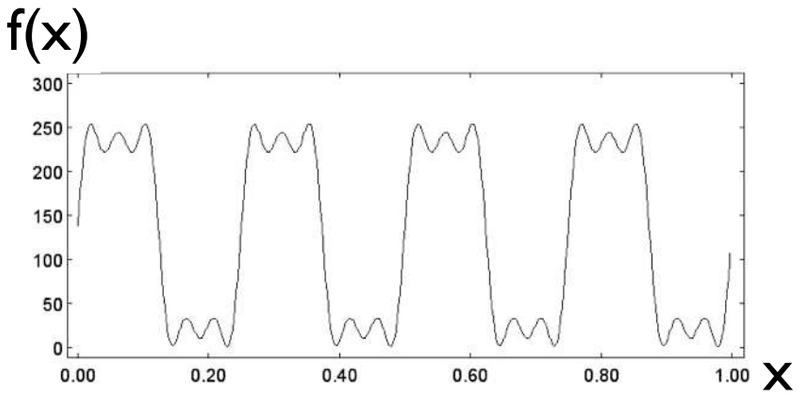


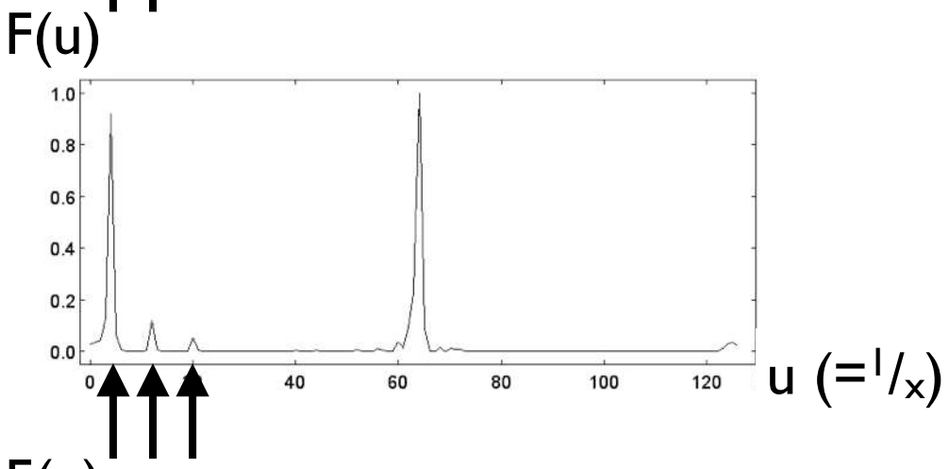

f(x)
x


F(u)
u (=1/x)

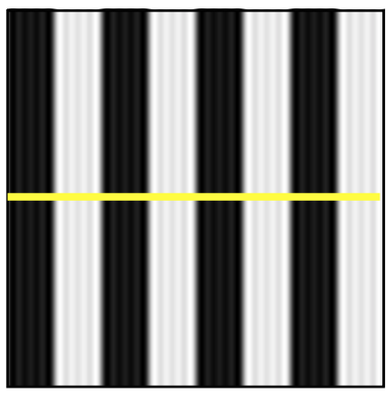
c

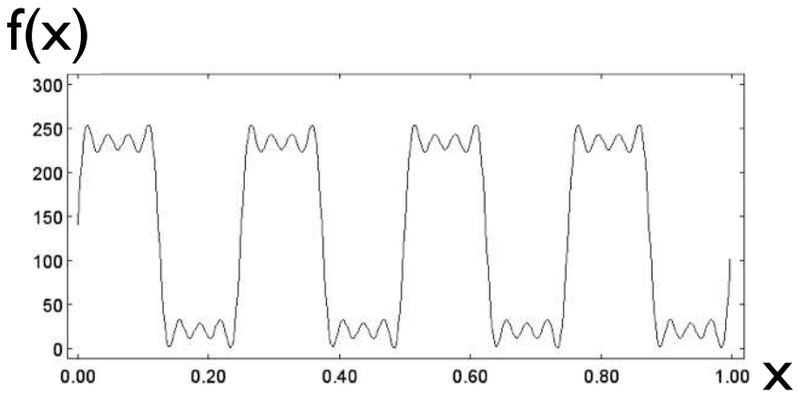


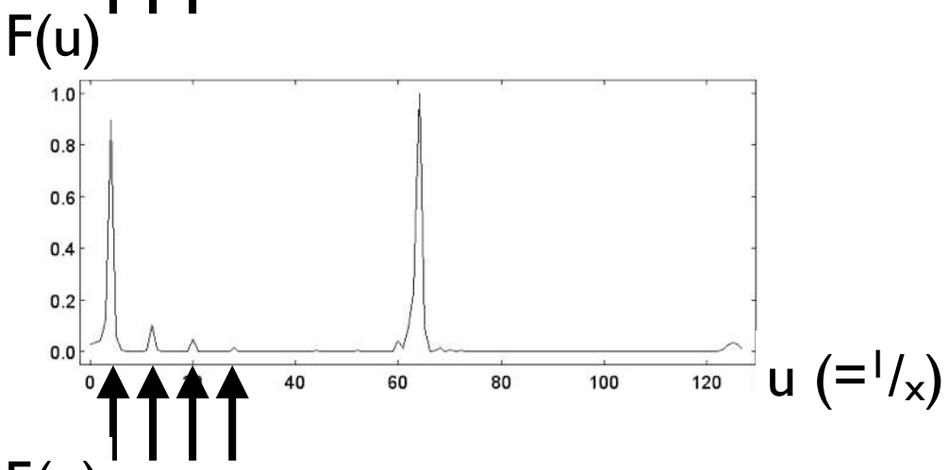

f(x)
x


F(u)
u (=1/x)

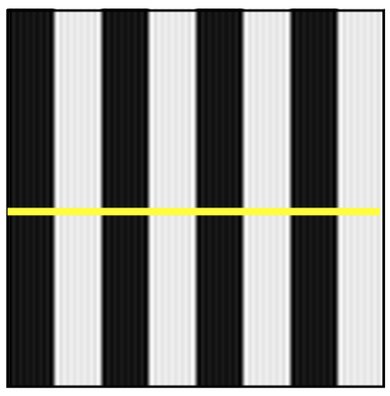
d

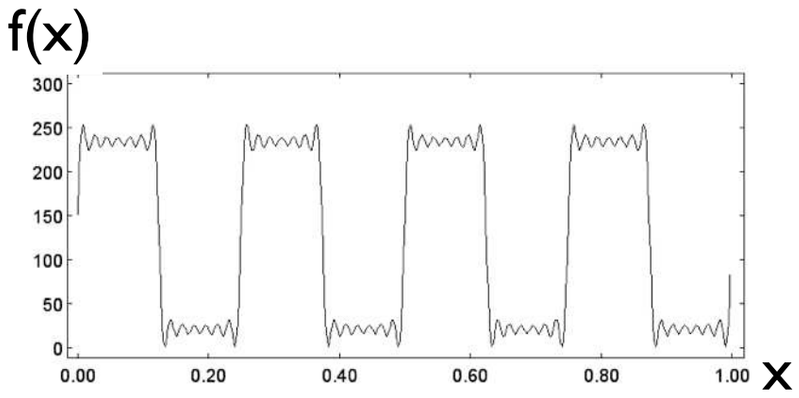


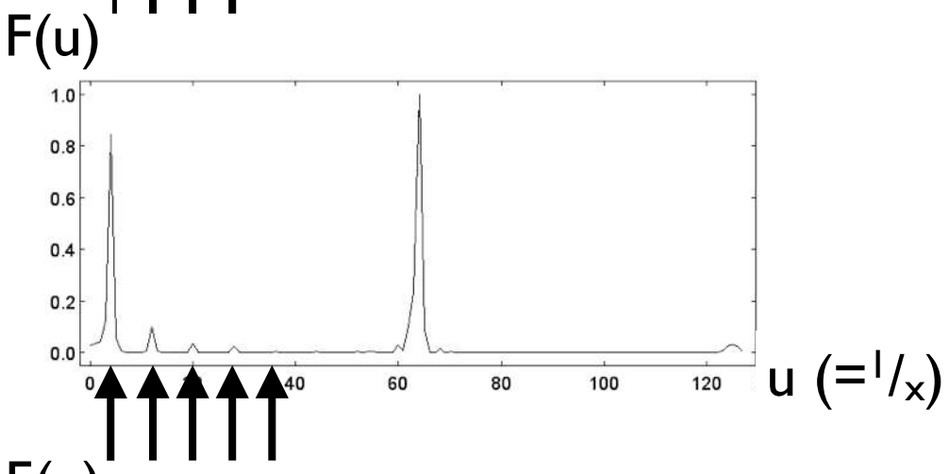

f(x)
x


F(u)
u (=1/x)

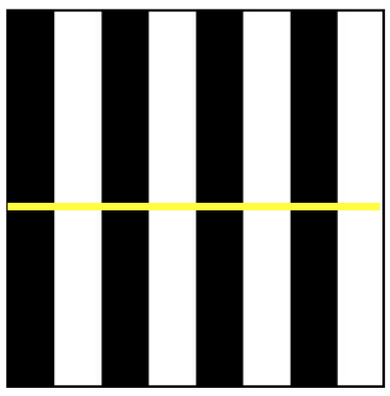
e

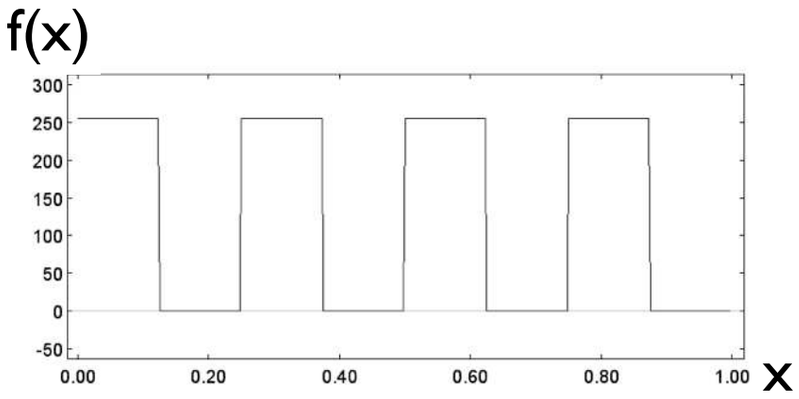


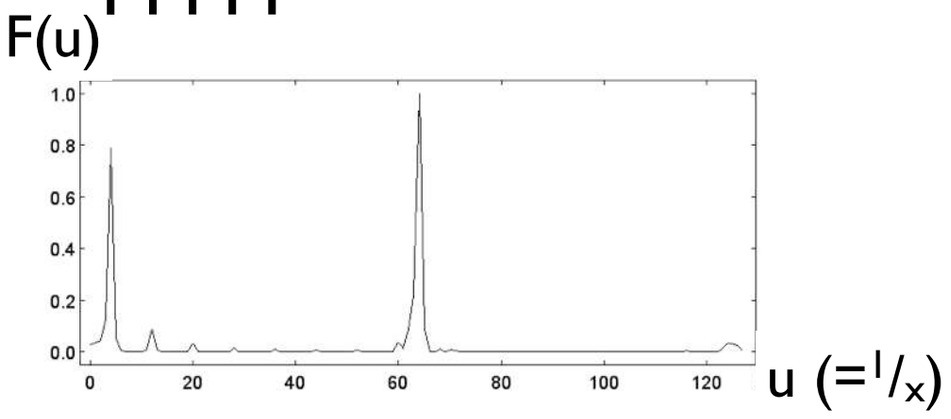

f(x)
x


F(u)
u (=1/x)

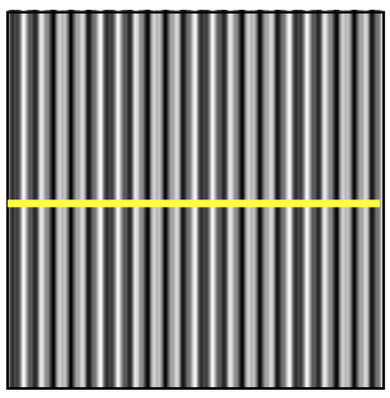
f

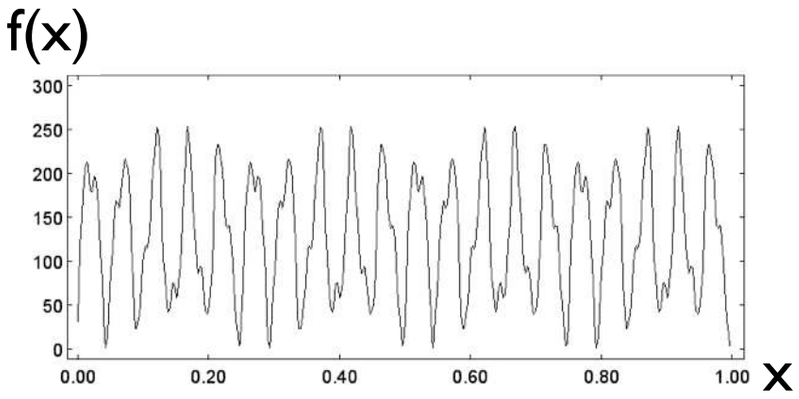


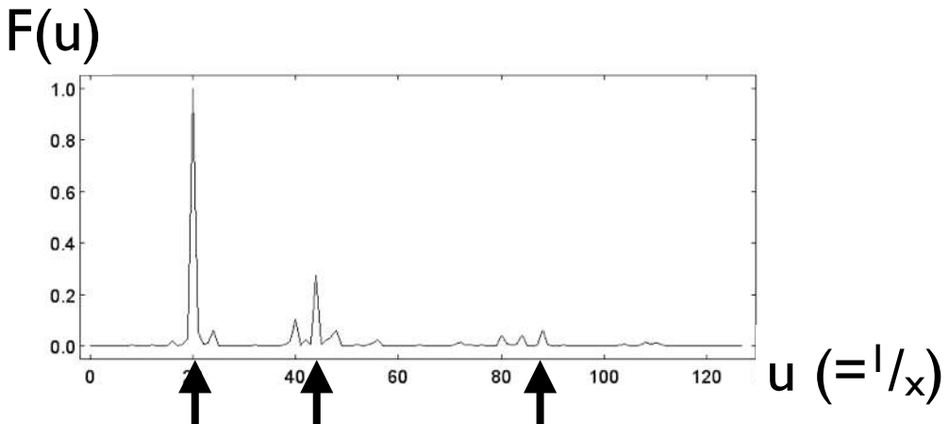

f(x)
x


F(u)
u (=1/x)

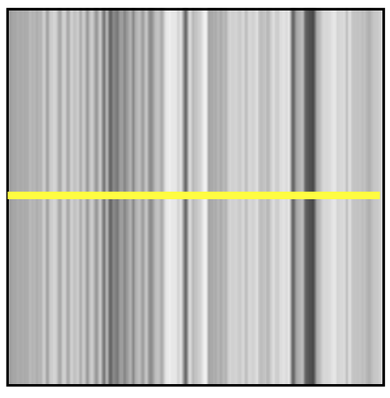
g

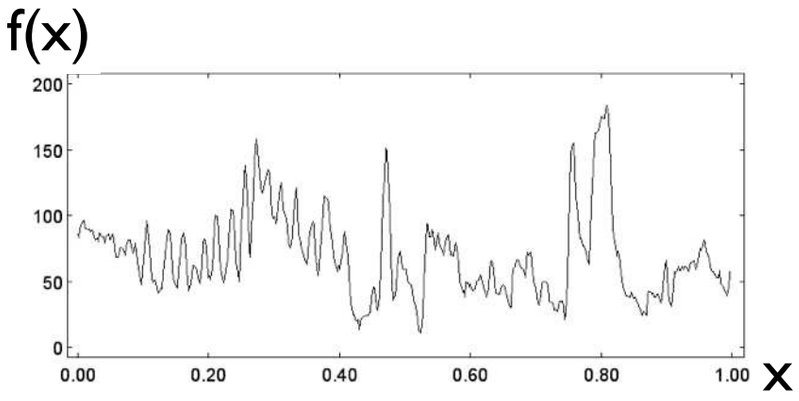



f(x)
x


F(u)
u (=1/x)

h




f(x)
x

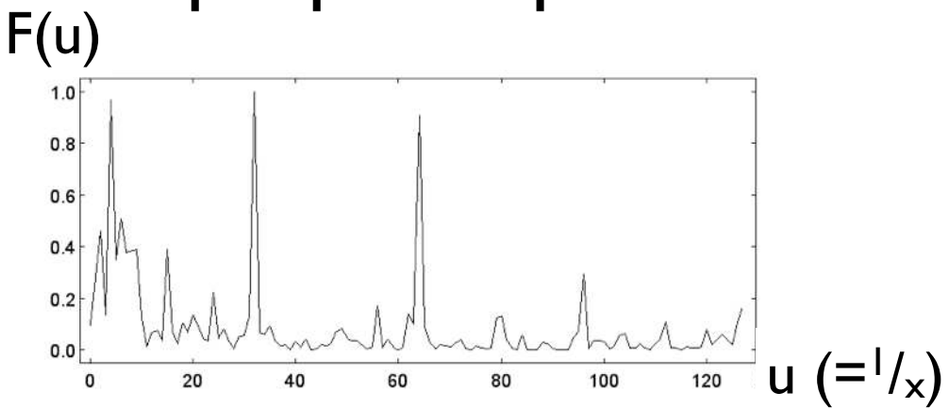

F(u)
u (=1/x)

Figure 19.4
One-dimensional Fourier Transforms.
From left to right: grayscale image, gray value profile, and Fourier transform shown for:
(a) fundamental sine wave, 4 per image width;
(b) fundamental plus 1 harmonic, 4 · 3=12 per image width;
(c) fundamental plus 2 harmonics, 4 · 5=20 per image width;
(d) fundamental plus 3 harmonics, 4 · 7=28 per image width;
(e) fundamental plus 4 harmonics, 4 · 9=36 per image width;
(f) rectangular wave;
(g) three superposed sine waves;
(h) general pattern;
gray value profile f(x) scaled to x =1 image width; Fourier transform F(u) with u = frequency per image width; note wrapping effect at u = 64 in FFT profiles.

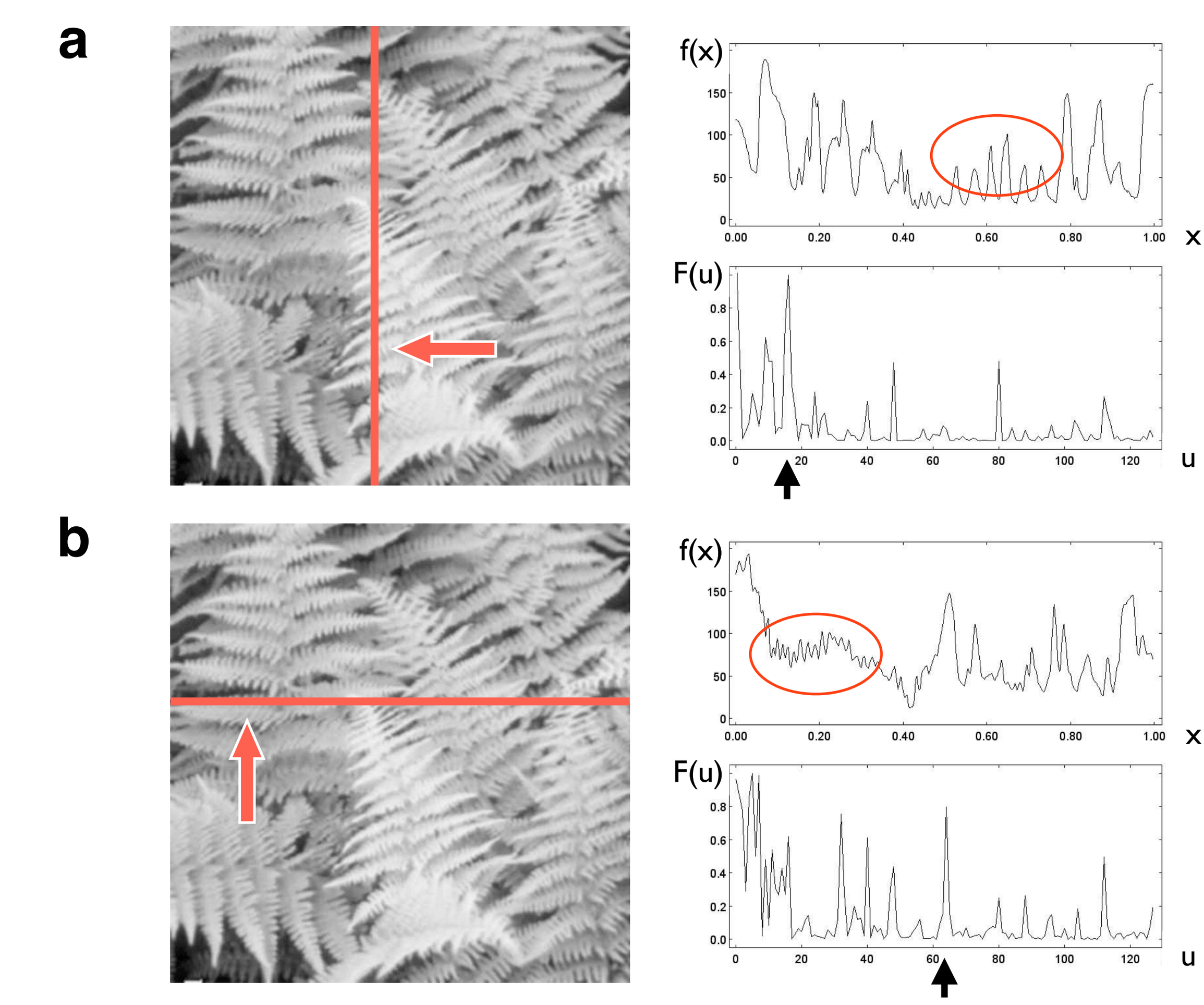


Figure 19.5
One-dimensional Fourier Transforms of a two-dimensional image.
(a) Image of fern with vertical trace, gray value profile and Fourier transform; short wavelengths are circled in f(x) plot (red) and located in image (red arrow), corresponding low frequencies are indicated in F(u) plot (black arrow);
(b) same as (a) with horizontal trace: here, short wavelengths, i.e., high frequencies are indicated;
gray value profile f(x) scaled to x =1 image width; Fourier transform F(u) with u = frequency per image width.

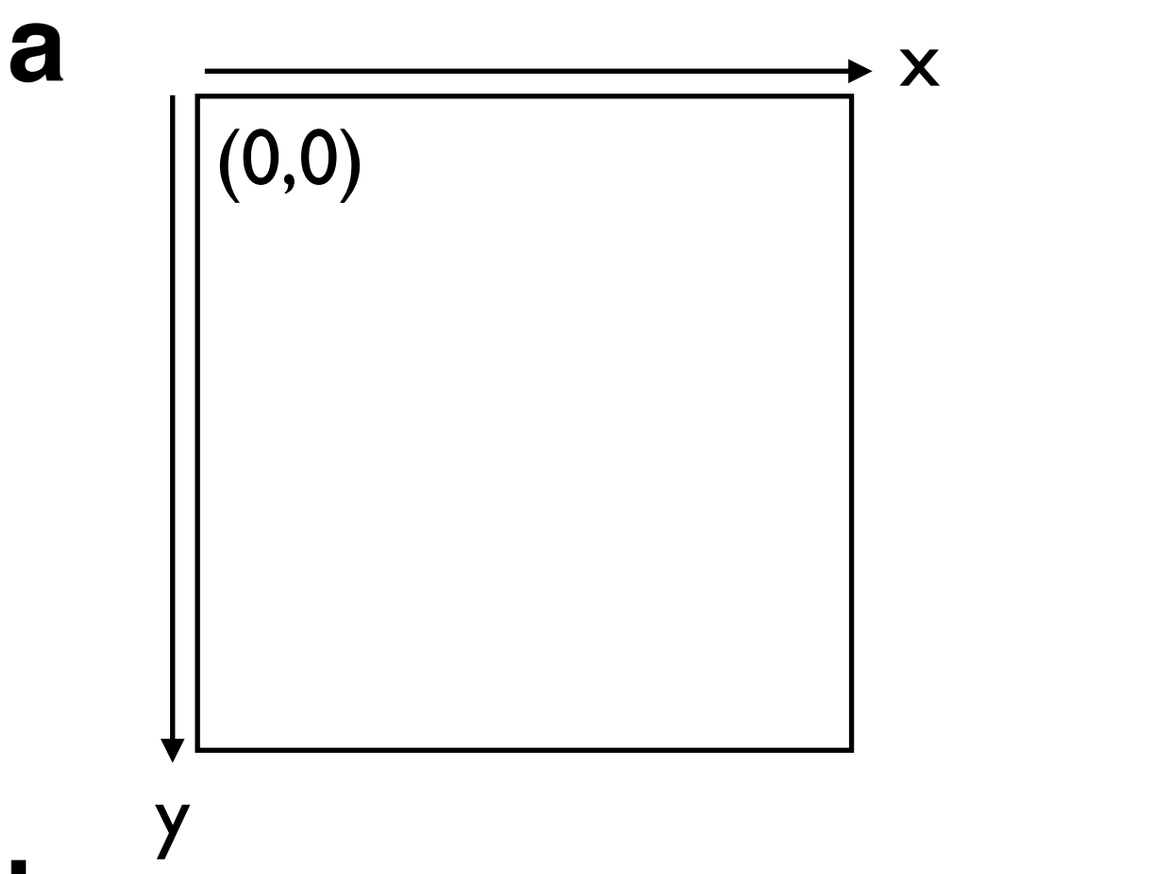


Figure 19.6
Coordinate systems.
(a) x-y coordinates of real image;
(b) u-v coordinates of Fourier transform image; note location of origin in center of image.

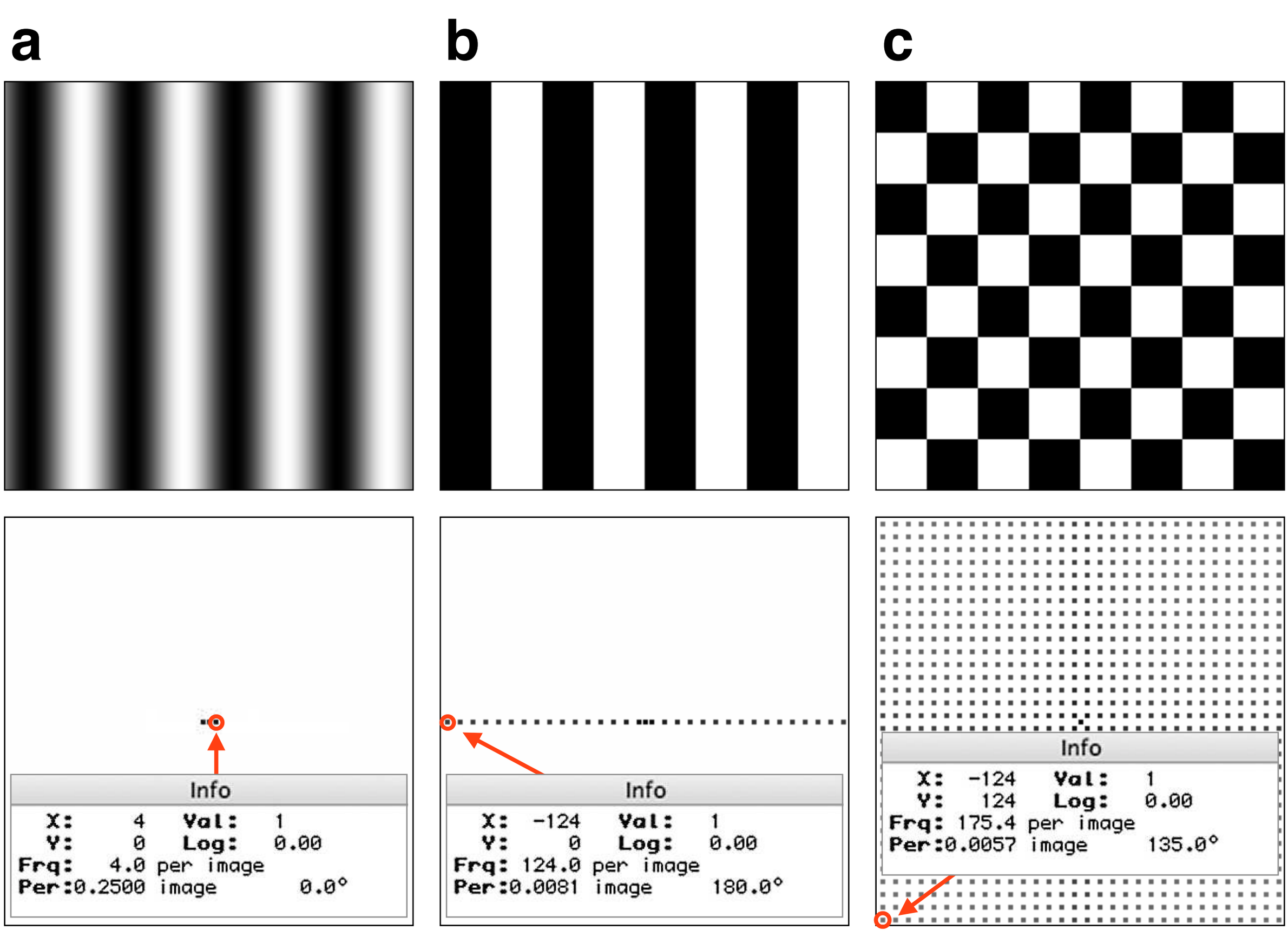


Figure 19.7
Two-dimensional Fourier Transforms of basic patterns.
Gray value images (top) and Fourier transforms (bottom) are shown for:
(a) horizontal sine wave;
(b) vertical black and white stripes;
(c) chess board;
the gray value images are scaled such that 256 pixels = 1 image width; selected frequencies are circle in red and corresponding Info window is shown.

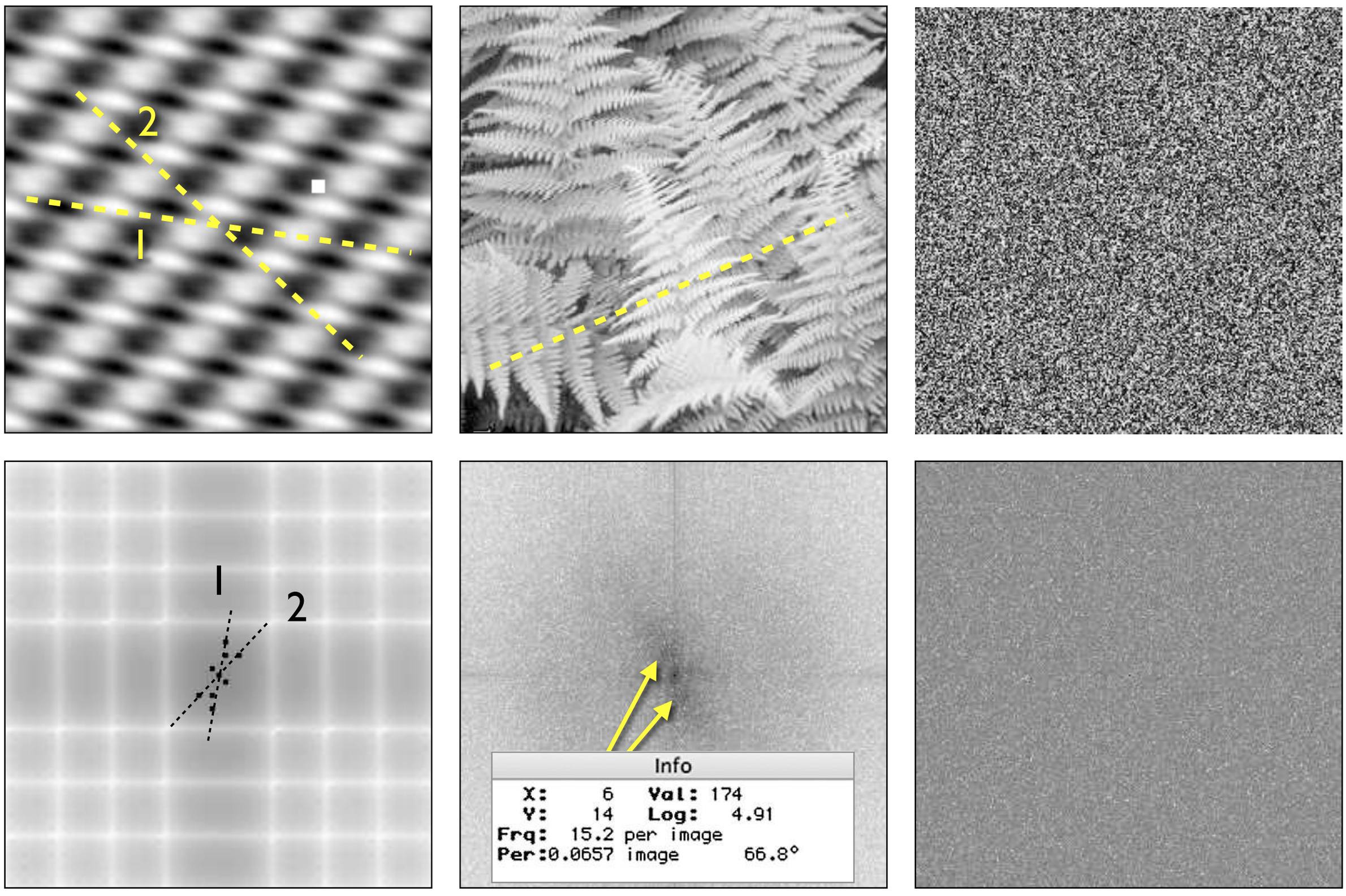


Figure 19.8
Two-dimensional Fourier Transforms of images.
Gray value images (top) and Fourier transforms (bottom) are shown for:
(a) superposition of four sine waves with little white cut-out square;
(b) fern;
(c) noise;
the gray value images are scaled such that 256 pixels = 1 image width; selected orientations and frequencies are highlighted.

Figure 19.9
Size of images and Fourier Transforms.
(a) Detail of 50 Euro bank note, image size is 2048 · 2048;
(b) Fast Fourier Transform (FFT) of (a), frames of 1024 · 1024 and 512 · 512 are indicated;
(c) FFT of 1024 · 1024 version of (a);
(d) FFT of 512 · 512 version of (a);
yellow arrows point at same high frequency spot in (b) and (c).

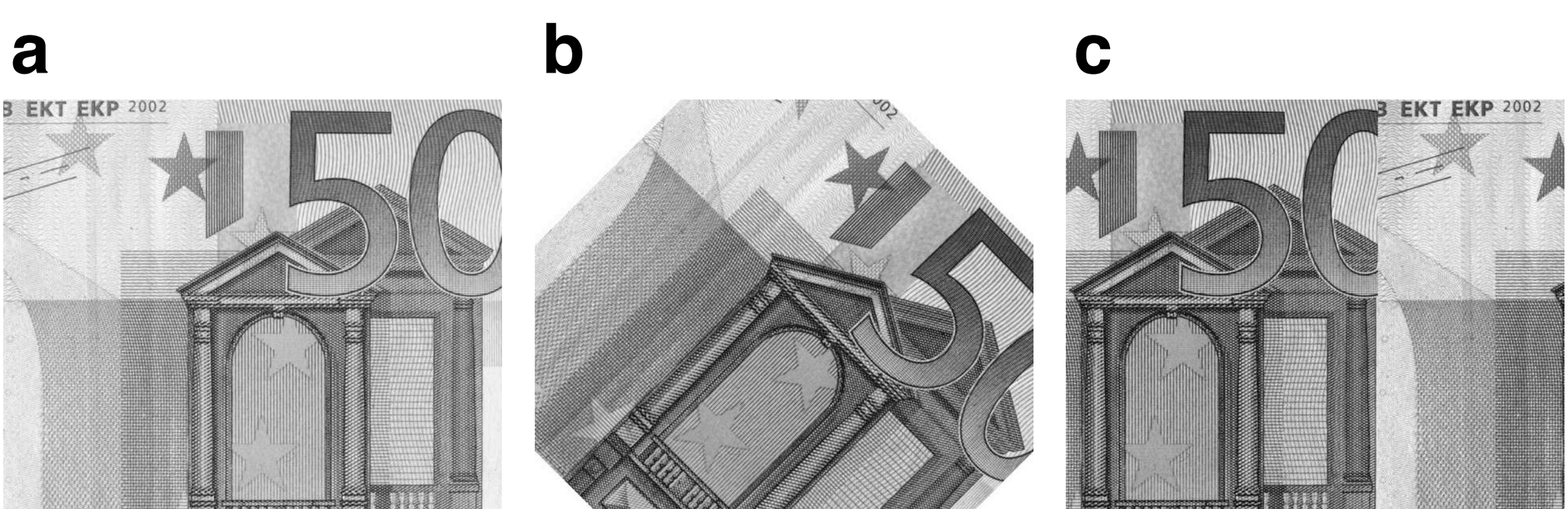


Figure 19.10
Orientation and displacement of images and Fourier Transforms.
Gray value image (top) and Fast Fourier transforms (bottom) are shown for:
(a) detail of 50 Euro bill;
(b) same as (a), rotated clockwise by 45°, Fourier transform is rotated;
(c) same as (a), cut in two, left and right piece swapped; Fourier transform is not affected.

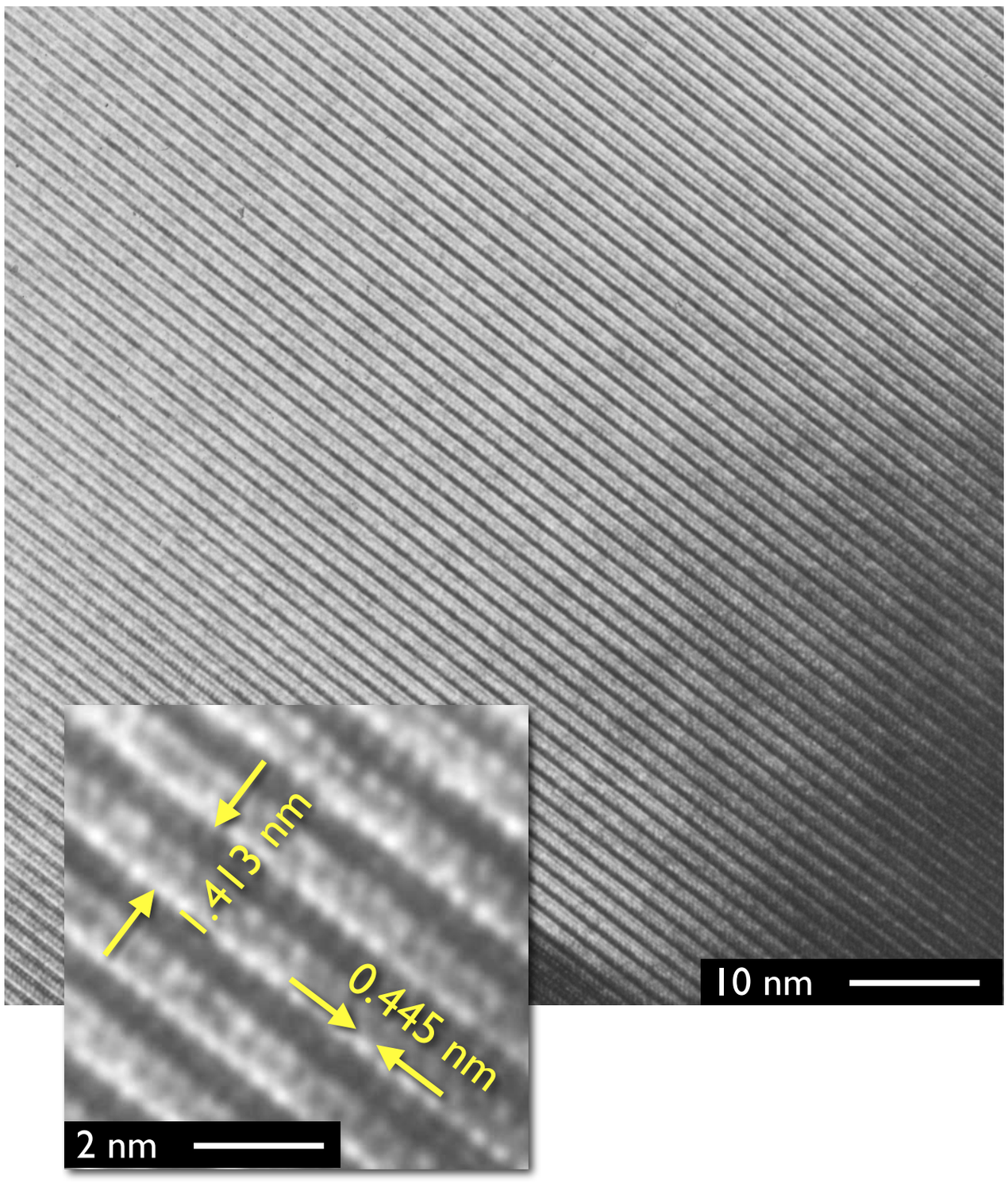


Figure 19.11
Transmission electron micrograph and Fourier Transform.
(a) Scaled high resolution image of chlorite crystal (image courtesy Andreas Kronenberg), image size 1024 · 1024, enlarged inset;
(b) Fourier transform of (a); two lattice spacings can be inferred.

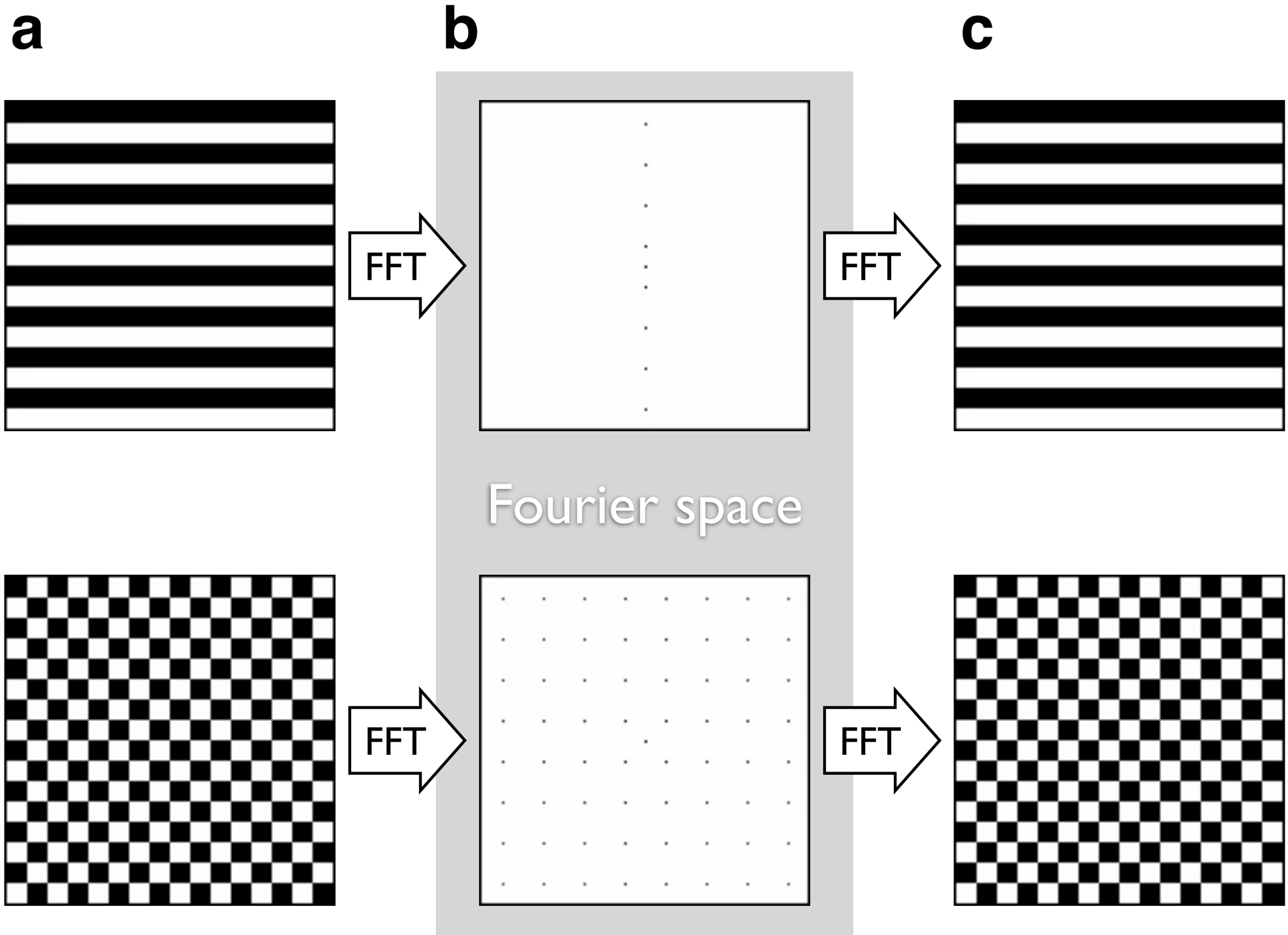


Figure 19.12
Forward and backward Fourier transforms.
(a) Original images;
(b) forward transform produces Fourier transforms of images;
(c) inverse transform reproduces original images from Fourier transforms.

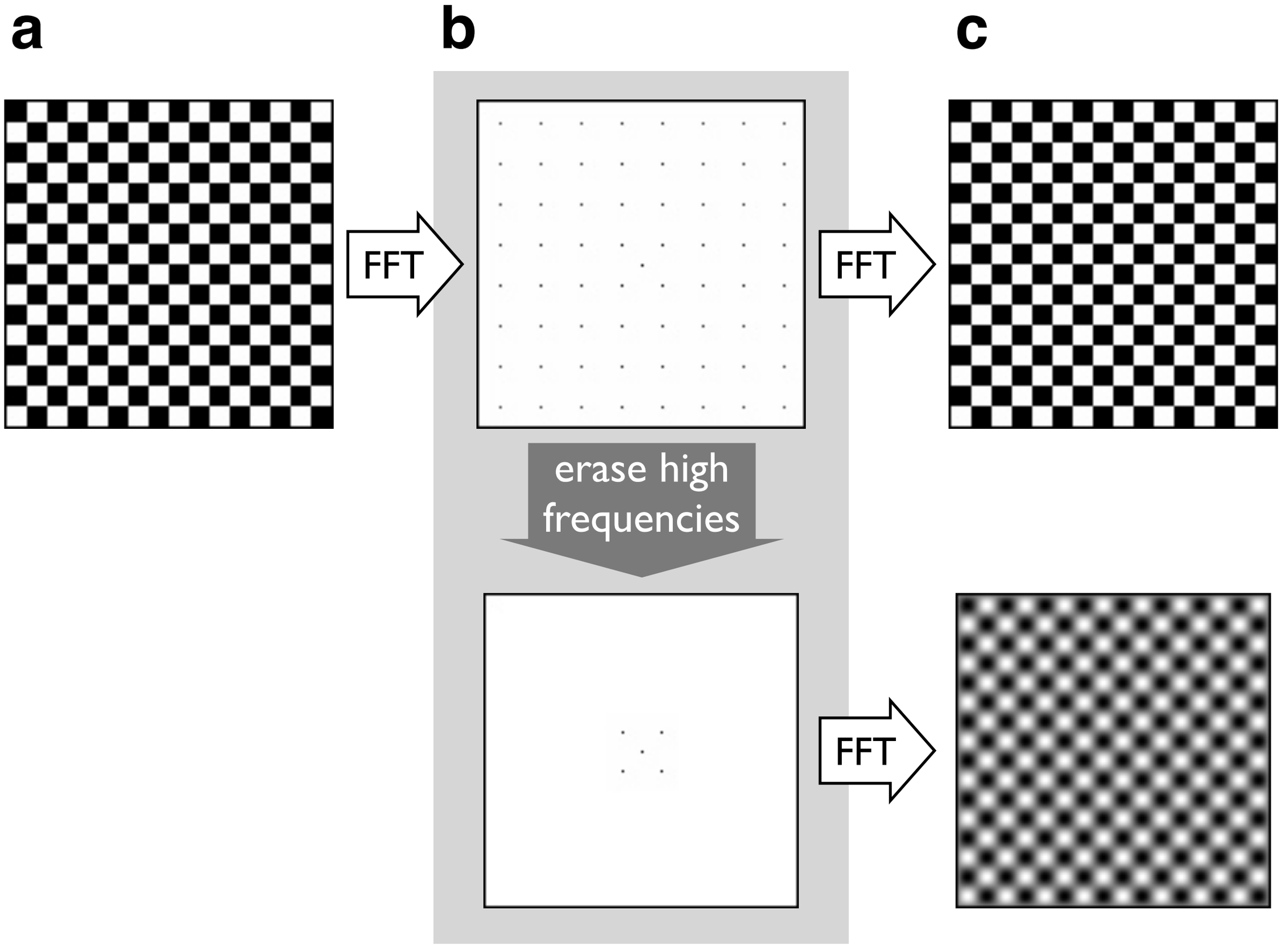


Figure 19.13
Modifying the Fourier transform.
(a) Original image (chess board);
(b) forward transform yields Fourier transform (top), high frequencies are removed (bottom);
(c) recalculated images from original and modified version of the Fourier transform.

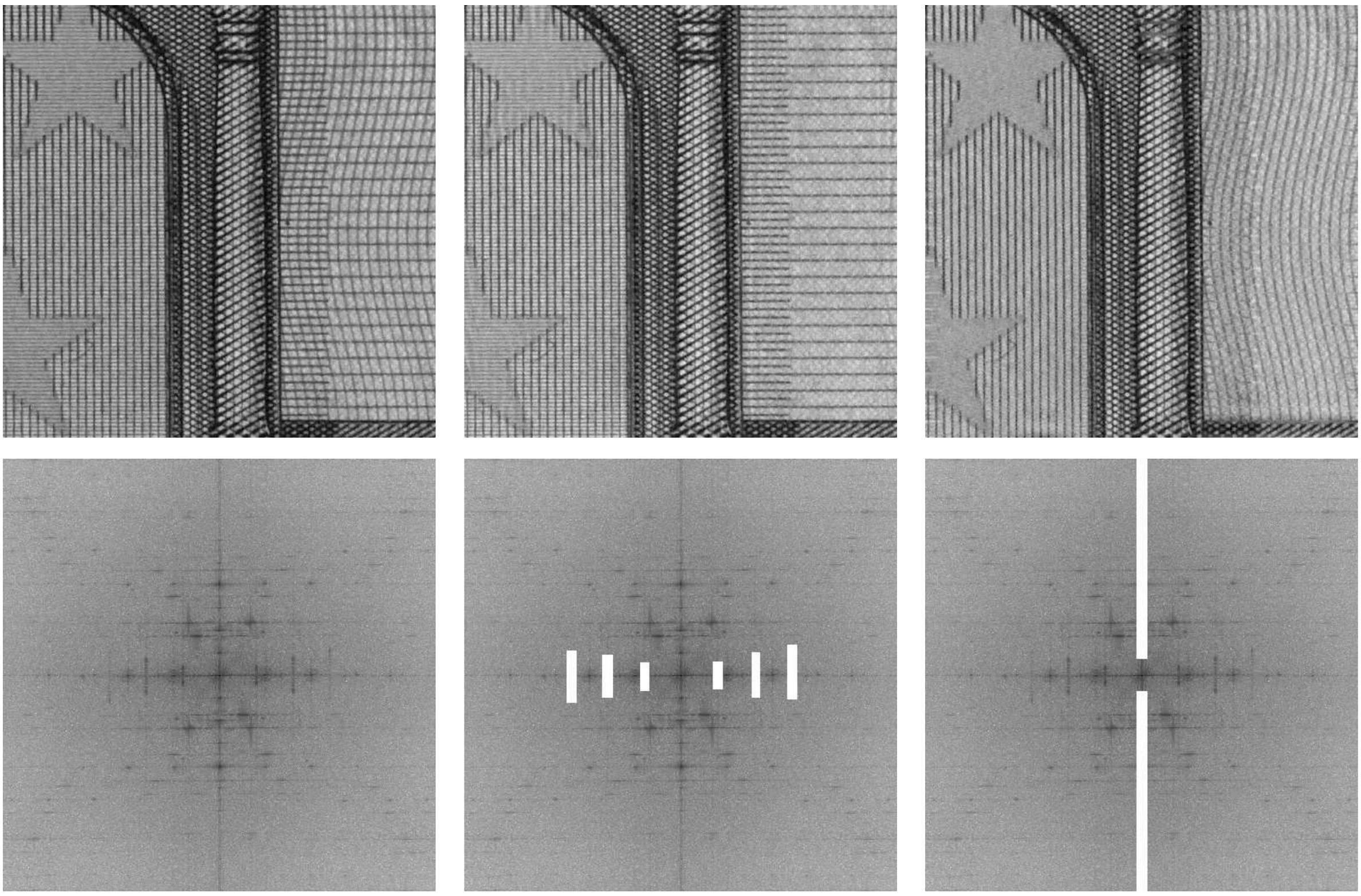


Figure 19.14
Suppressing selected rulings in printed material.
(a) Detail of 50 Euro bank note (top) and Fourier transform (below); image size 1024 · 1024;
(b) horizontal frequencies, belonging to wavy vertical ruling (right side of image) removed;
(c) vertical frequencies belonging to horizontal ruling (right side of image) removed.

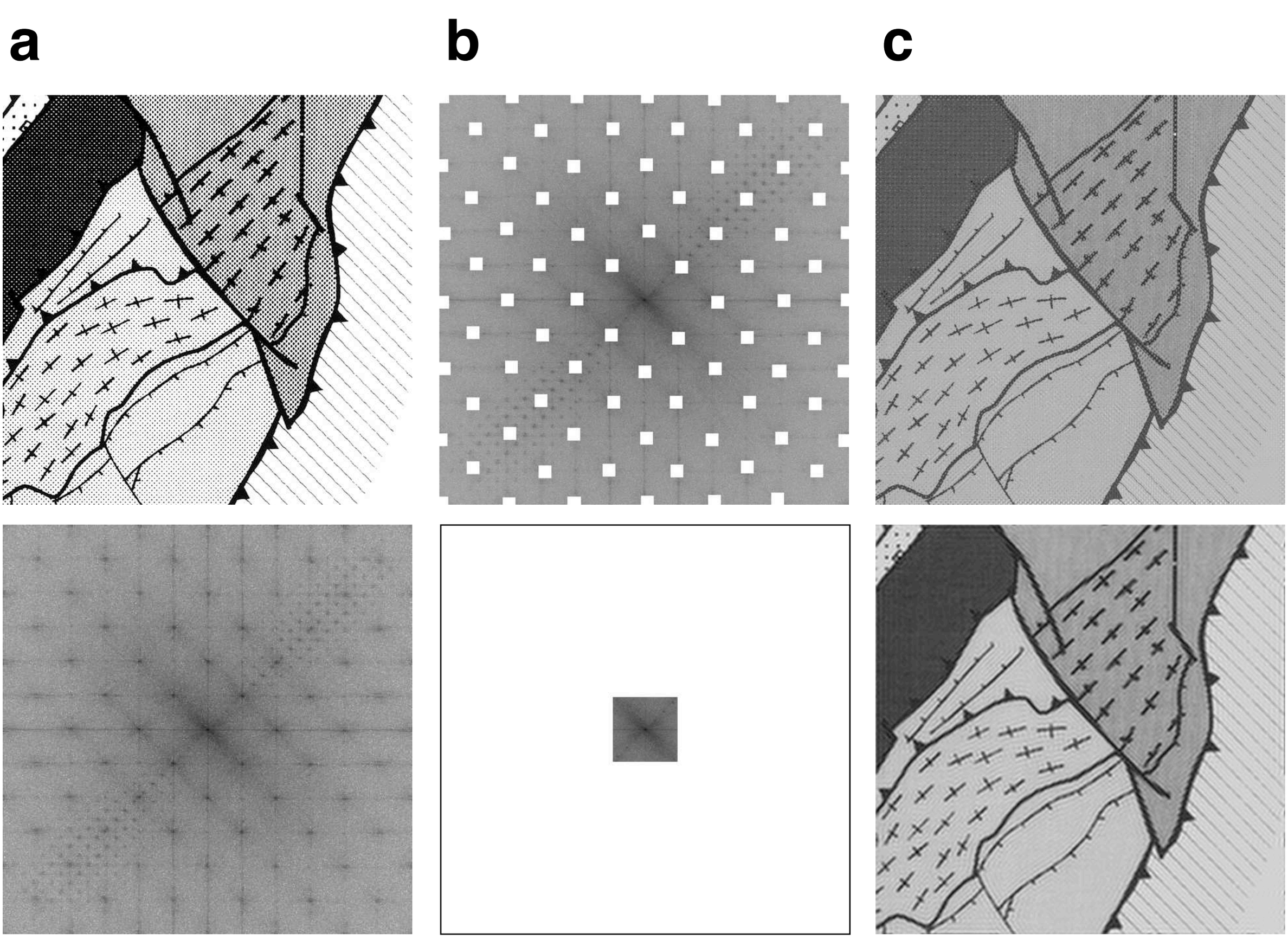


Figure 19.15
Frequency filtering of printed map.
(a) Detail of a tectonic map, showing different rulings for different structural units (top), FFT (bottom); image size is 1024 · 1024;
(b) selected frequencies are erased (top) all but the lowest frequencies are removed (bottom);
(c) inverse FFT of modified Fourier transforms (b).

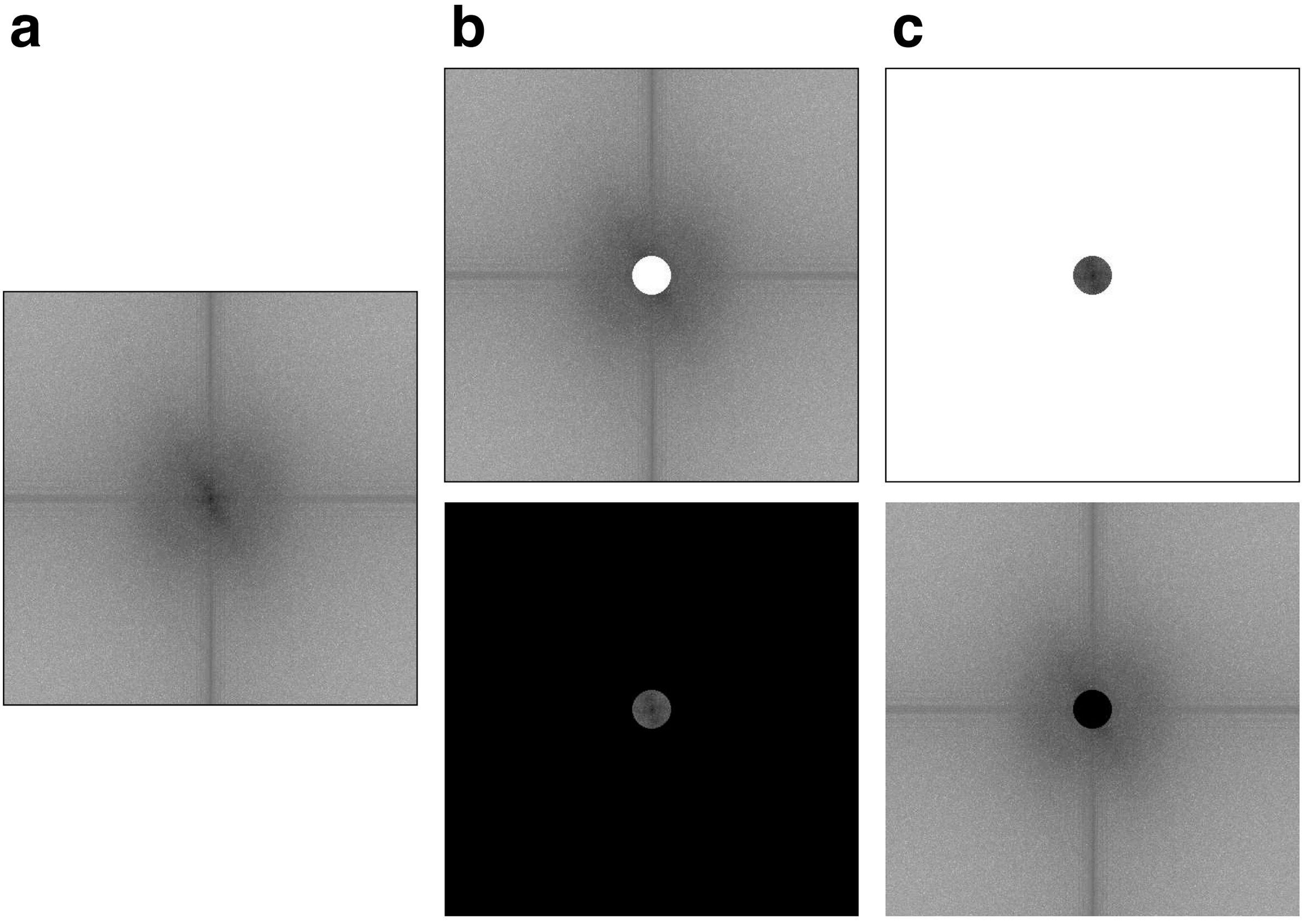


Figure 19.16
Creating Fourier filters in Image SXM.
(a) Fourier transform of image (image = fern, as shown in Figure 19.8.b);
(b) high-pass filtering either by blocking ('erasing') low frequencies (white, top) or by passing ('painting') high frequencies (black, bottom);
(c) low-pass filtering either by blocking ('erasing') high frequencies (white, top) or by passing ('painting') low frequencies (black, bottom).

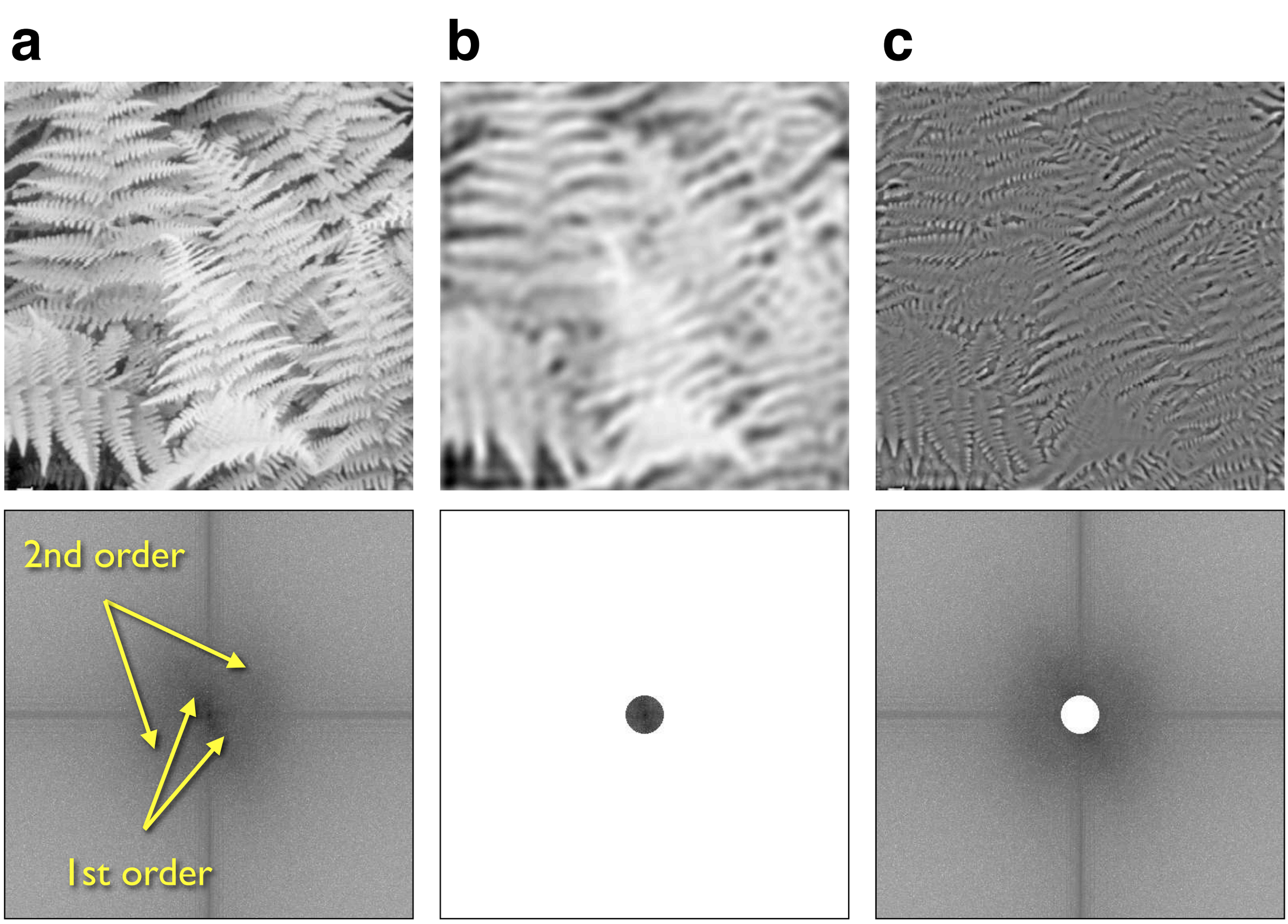


Figure 19.17
High and low-pass filtering of image.
(a) Image of fern (top) with Fourier transform (bottom); arrows indicate frequencies of 1st and 2nd order frequencies;
(b) low-pass filtering of (a);
(c) high-pass filtering of (a);
image size is 512 · 512, the radius of the filter mask is 24 pixels.

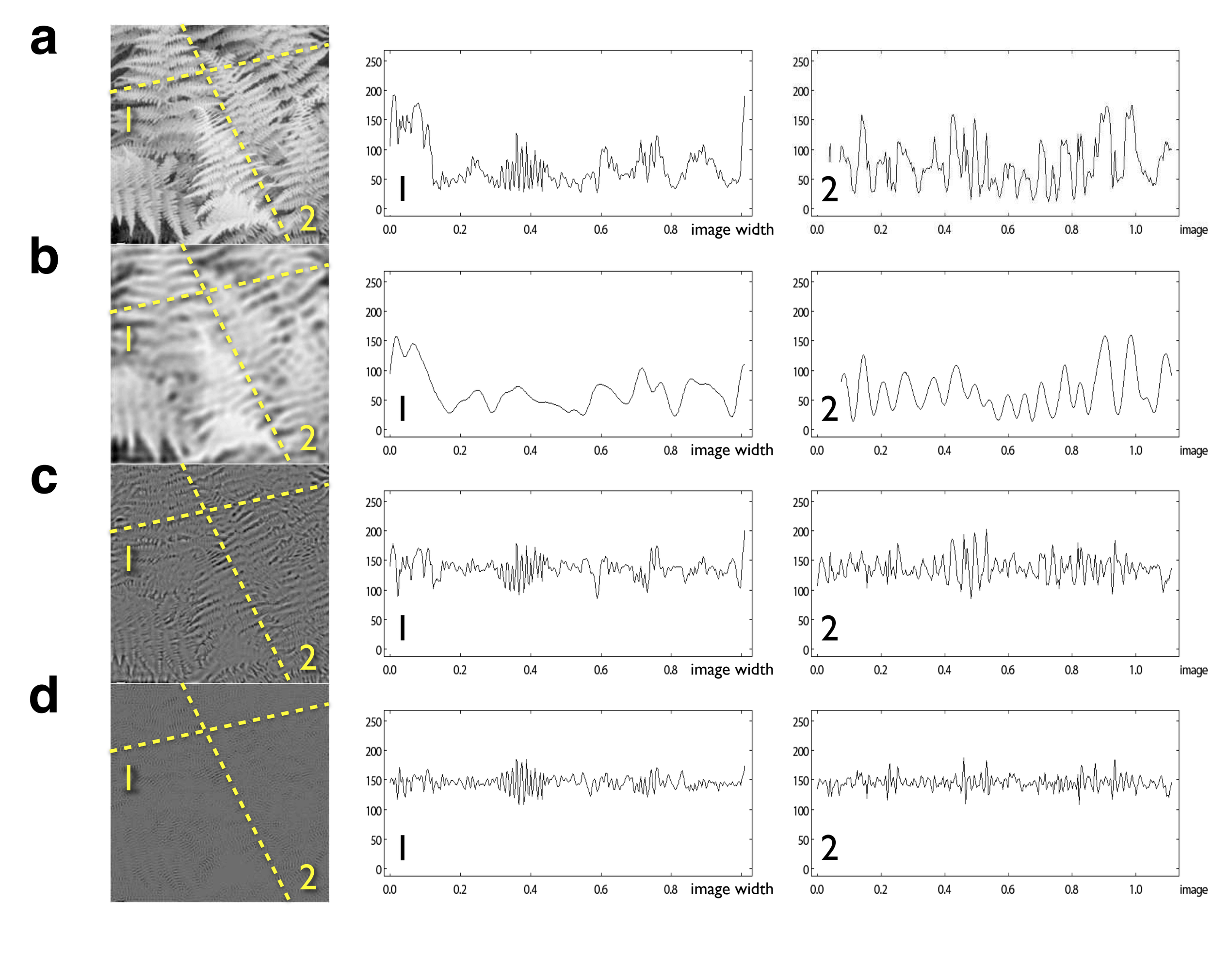


Figure 19.18
Frequency content of image.
(a) Image of fern (left) and gray value profiles (right); traces of profiles are indicated;
(b) low-pass filter: frequencies < 24 per image;
(c) high-pass filter: frequencies > 24 per image;
(d) ultra high-pass filter: frequencies > 48 per image.

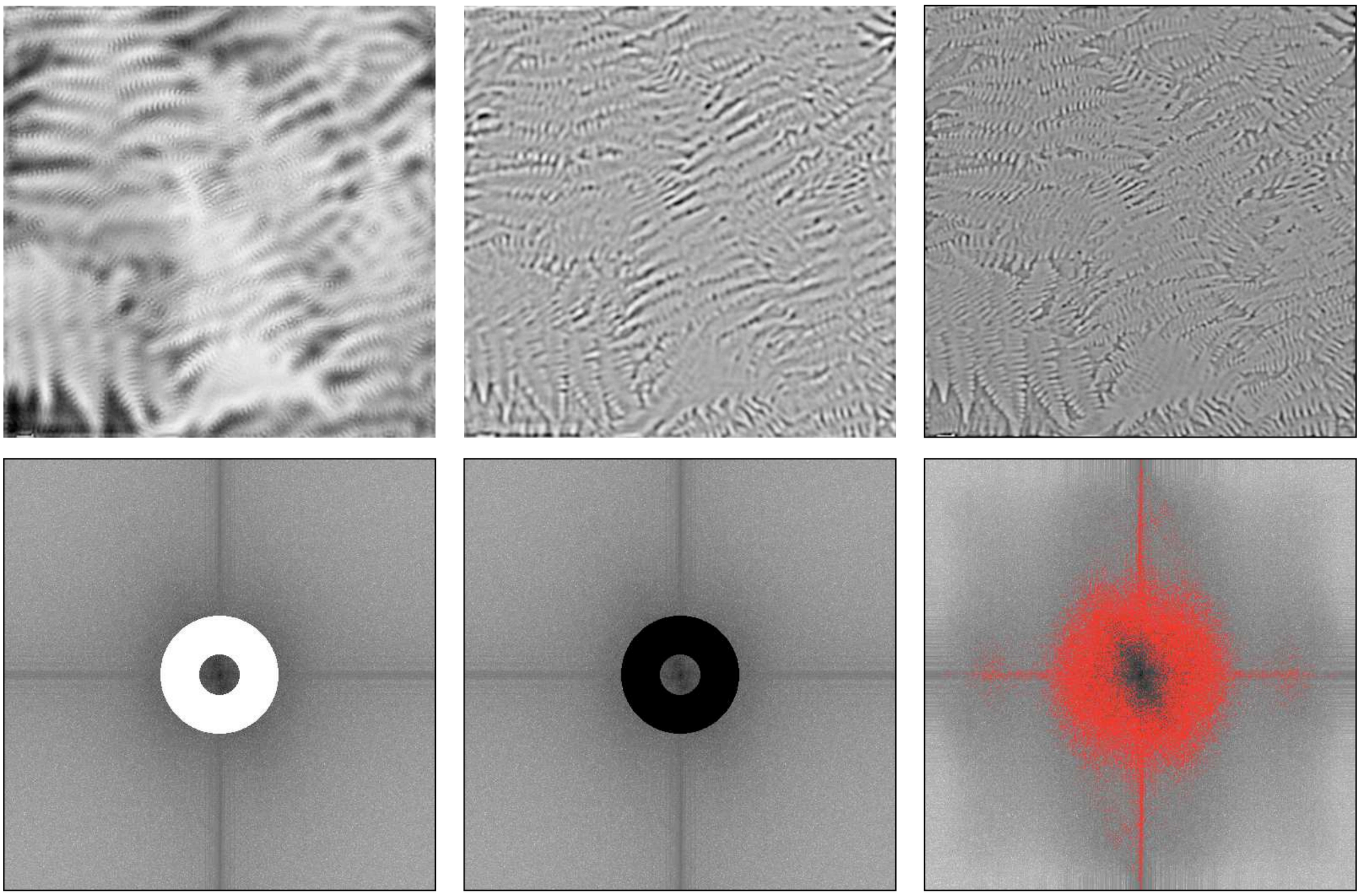


Figure 19.19
Frequency slicing.
Images after filtering (top) and corresponding frequency filters (bottom); original image = fern (Figure 19.18), image size is 1, frequencies are given per image.
(a) Pass filtering for (frequency < 20) and (frequency > 70);
(b) pass filtering for (20 < frequency < 70);
(c) pass filtering for frequencies with values (132 < gray value < 164).

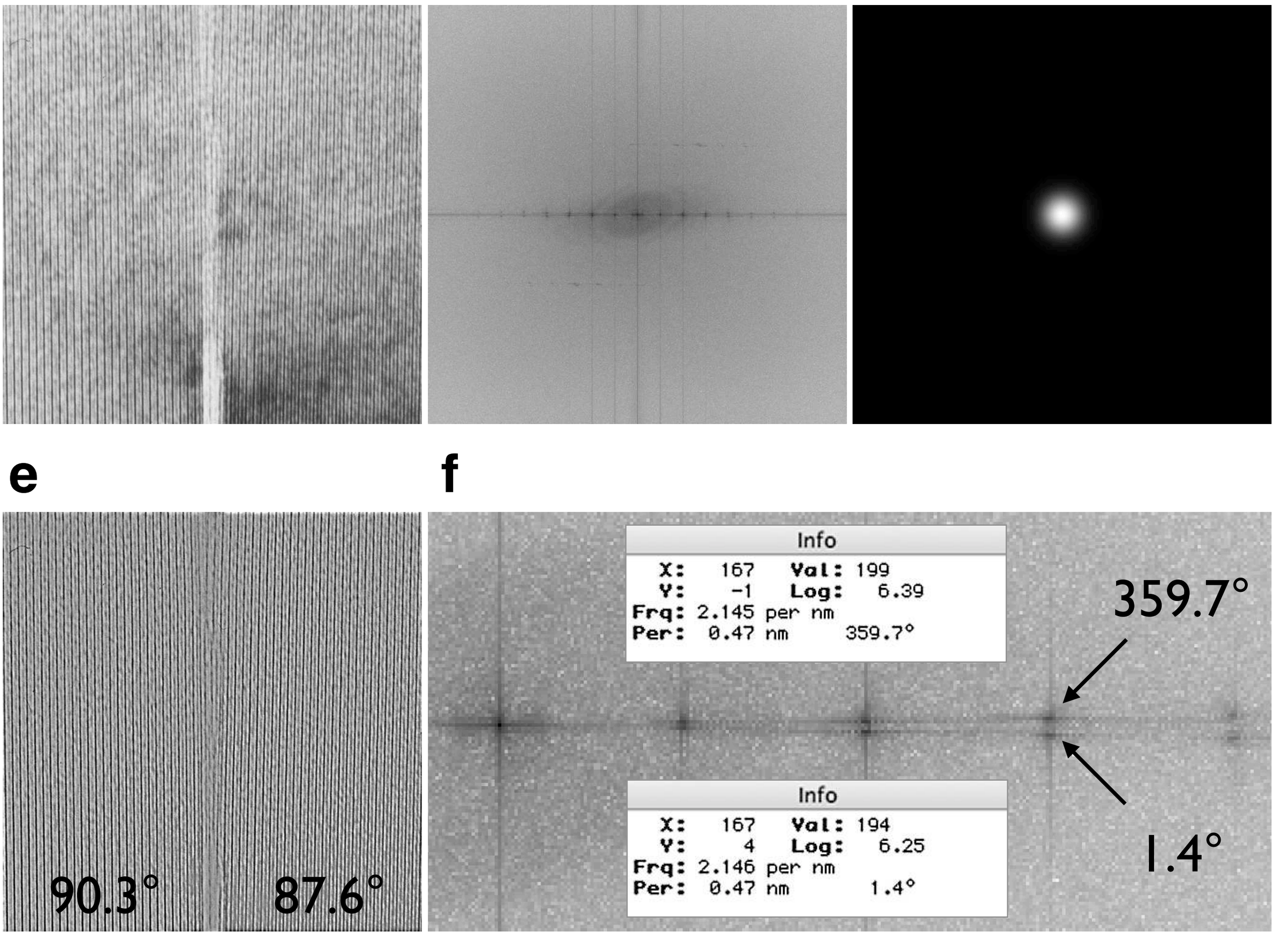
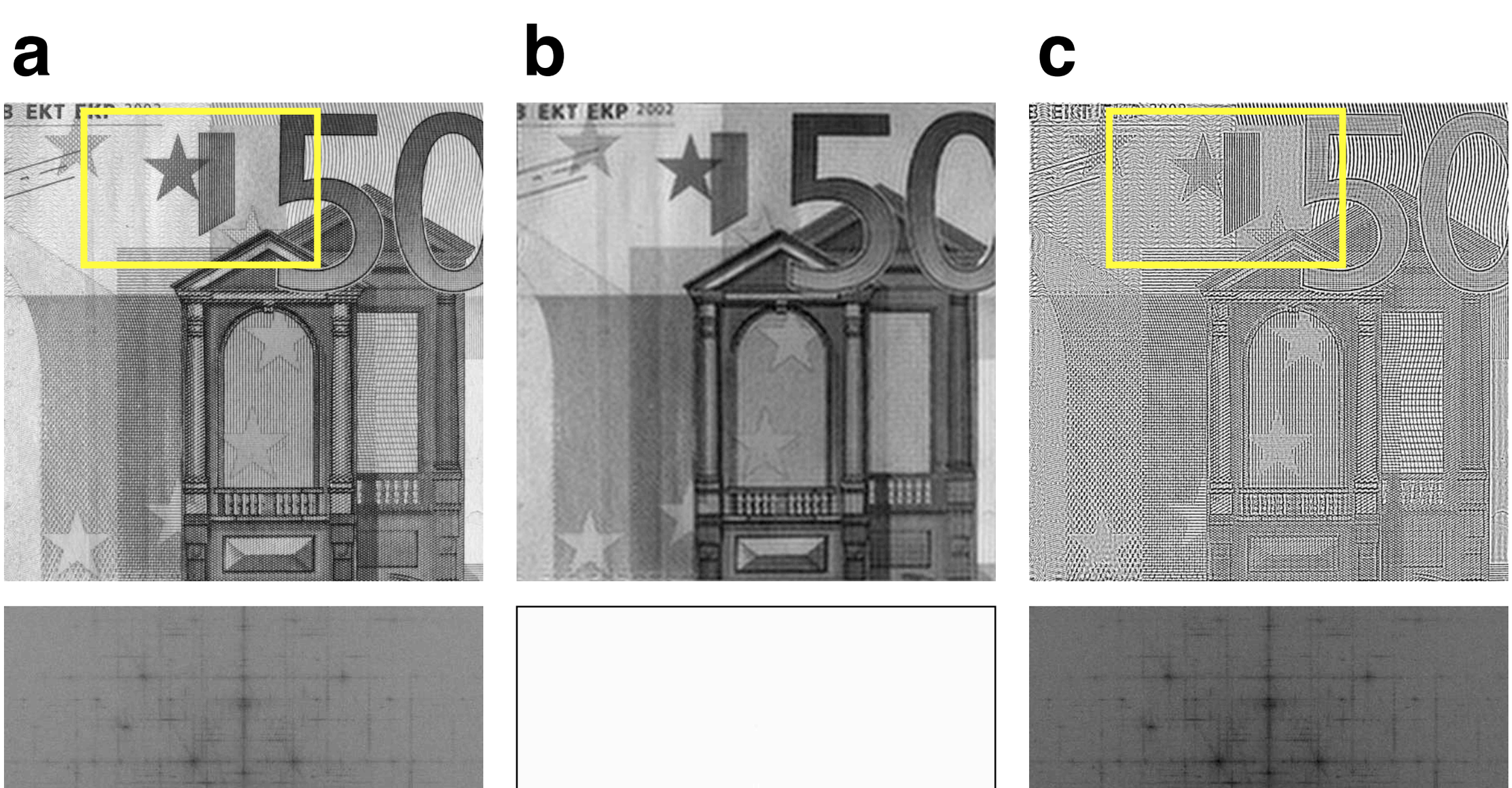


Figure 19.20
Analysis of crystal lattices.
(a) High resolution TEM micrograph showing tilt wall in chlorite crystal;
(b) Fourier transform of (a);
(c) high-pass filter used for (c); black = pass, white = blocked ('erased'); filter width = 20% of image width, transition length of filter = 100% of filter width;
(d) filtered version of (a) using mask shown in (c);
(e) identification of orientation of lattice on left and right of tilt wall shown.



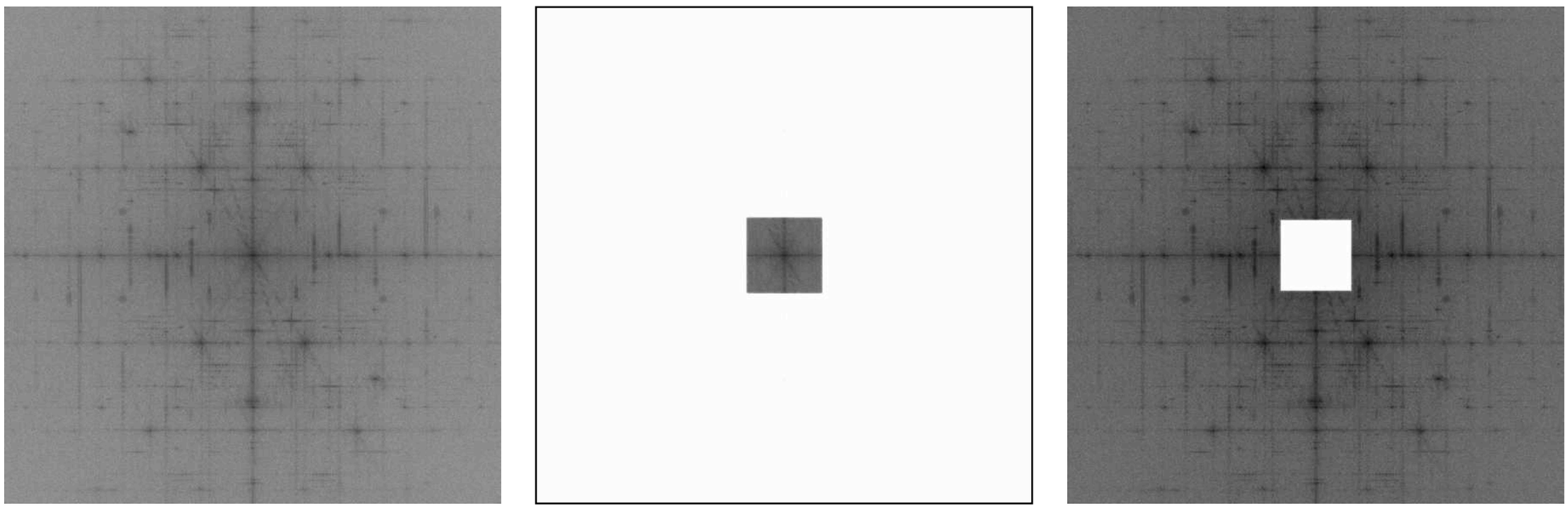


Figure 19.21
Edge detection with Fourier filtering.
(a-c) Detail of 50 Euro bank note (above) and Fourier transform (below);
(a) Original:
(b) low-pass filtering, filter width = 15% of image width;
(c) high-pass filtering, using same filter size as (b);
(d) enlarged view of detail outlined by yellow frame in original (a) and after thresholding (below);
(e) enlarged view of detail outlined by yellow frame in high-pass filtered version (c) and after thresholding (below).

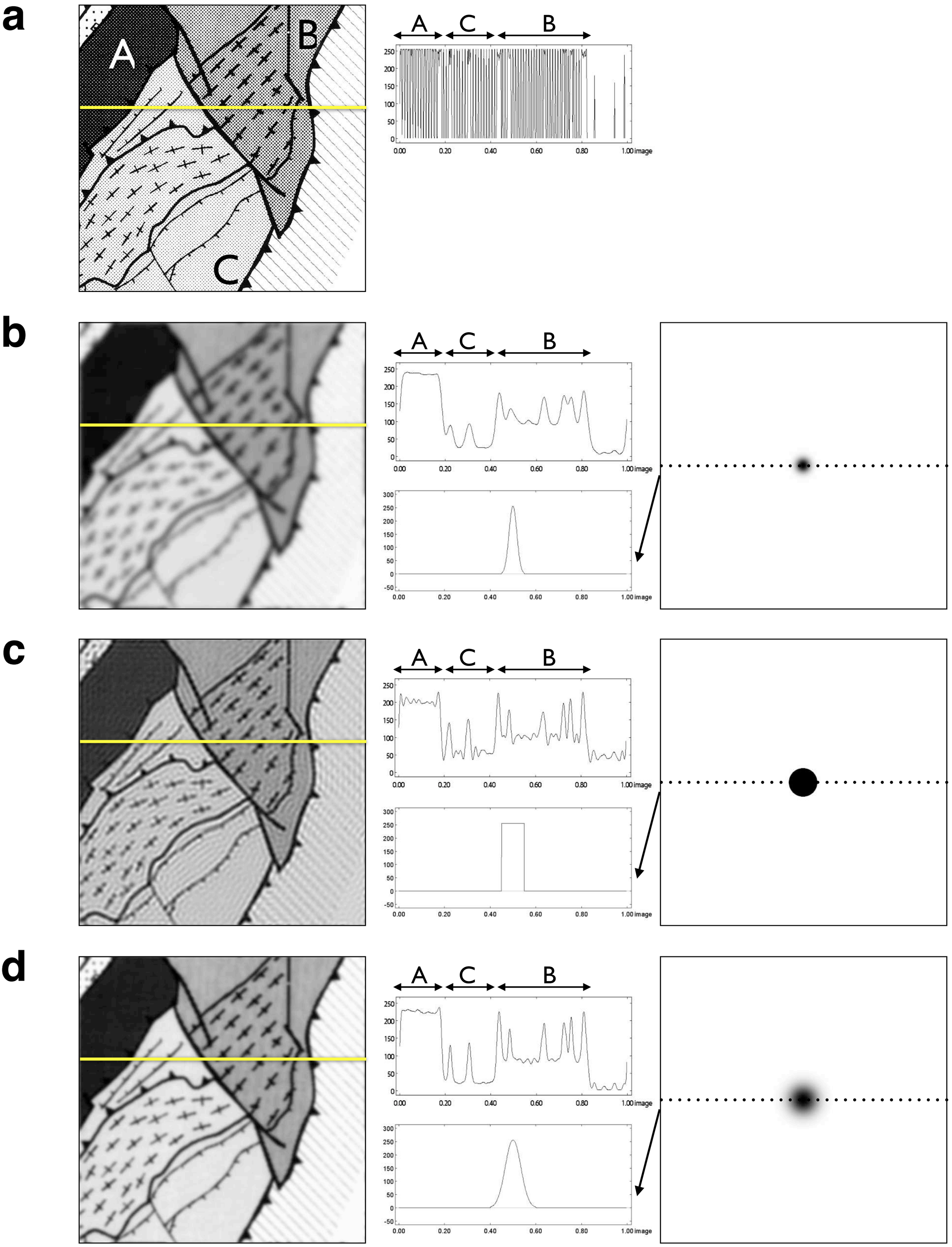


Figure 19.22
Creating halftone images through Fourier filtering.
(a) Original drawing (tectonic map, see Figure 19.15.a); gray value profile; trace of profile is indicated; note different hatching systems; units A, B, C are indicated;
(b) to (d) low-pass filtered versions of (a) using different filter sizes and transition widths; filter mask is shown on right; gray values profile and profile of filter are shown in center;
(b) filter width = 10 % of image; transition width = 100% of filter width;
(c) filter width = 10 % of image; transition width = 0% of filter width;
(d) filter width = 20 % of image; transition width = 100% of filter width.